Civil Engineering

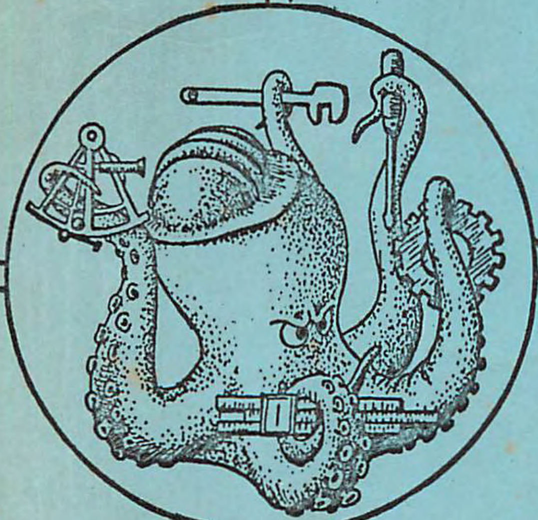
201 A

Physical

Oceanology

Pat Wilde - Instructor

COMMITTEE ON OCEAN ENGINEERING
COLLEGE OF ENGINEERING



UNIVERSITY OF CALIFORNIA
BERKELEY

Fall Quarter

UNIVERSITY OF CALIFORNIA
CE 201A, Fall

Department of Civil Engineering
Div. of Hydraulic and Sanitary Engr.
Instructor: P. Wilde

Tentative Lecture Schedule

Week	
	Physical Parameters
1	Fundamental Oceanographic Parameters: Position, depth, temperature, pressure and salinity
2	Derivation of other Oceanographica Parameters from thermodynamic considerations
3	Measurements at sea, Z, T, S, Z-T, and T-S graphs, concepts of water masses
	Fluid Mechanics
4	Accelerations on a rotating sphere, Coriolis effect - Geostrophic assumption cyclonic - anticyclonic flow
5	Non frictional assumptions - Margules and Helland-Hansen equations
	Oceanic Circulation
6	Air-Sea interactions - atmospheric effects - wind belts, barotrophic baroclinic ocean - monsoonal currents
7	Synoptic oceanic circulation
8	Dynamic topography - depth of no motion - transport calculations
9	Frictional assumptions - Ekman circulation - ocean models
	Applied Physical Oceanography
10	ASW, marine pollution control, marine waste disposal, new instruments, potential new developments

Physical Oceanography

201 A

Fall - '73

Department of Civil Engineering
Div. of Hydraulic and Sanitary Engr.
Insyructor: Pat Wilde

Basic Reading List

Fluid Mechanics on a Rotating Body:

Defant	Chap. X, XI, XIII
Proudman	I, II, III, IV
S. J. F.	~~XII~~
Von Arx	4

Physical Properties of Sea Water:

Defant	Chap. II, III, IV, V, VI
Dietrich	II, III, V
Pickard	3, 5
Proudman	VII
S. J. F.	III, IV, X
Von Arx	5

Oceanic Circulation:

Defant	Chap. XVIII, XX, XXI
Dietrich	X
Pickard	4, 7
S. J. F.	XV
Von Arx	6, 7

Applications of Mathematical Models to Understanding Oceanic Circulation:

Defant	Chap. IX, XV, XVII
Dietrich	VII
PROUDMAN	IV, V, VII, IX
Robinson (ed)	All Reprints
S. J. F.	XI, XIII
Von Arx	8, 9, 10, 11

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201A Fall

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PHYSICAL OCEANOLOGY BIBLIOGRAPHY

Books:

Bertschneifer, C. L., 1963, OCEAN WAVE SPECTRA: The Symposium sponsored by the National Academy of Science, 357 p.

Cotter, C. H., 1966, THE PHYSICAL GEOGRAPHY OF THE OCEANS: New York, American Elsevier Pub. Co., Inc. 317 p.

Dietrich, G., 1963, GENERAL OCEANOGRAPHY: New York, Interscience - Wiley, 588 p.

Defant, A., 1961, PHYSICAL OCEANOGRAPHY: New York, Pergamon Press, v. I, 929 p., v. II, 598 p.

Eckart, C. H., 1960, HYDRODYNAMICS OF OCEANS AND ATMOSPHERES: London, Pergamon, 290 p.

Fairbridge, R., 1966, THE ENCYCLOPEDIA OF OCEANOGRAPHY: New York, Reinhold, 1020 p.

Fomin, L.M., 1964, THE DYNAMIC METHOD IN OCEANOGRAPHY: Elsevier Oceanography Series 2 (U.S.S.R. Academy of Sciences, Moscow), 212 p.

Hill, M.N. (ed.) 1962, THE SEA: PHYSICAL OCEANOGRAPHY - VOL I: New York, John Wiley and Sons, 864 p.

Hunt, L.M. and D. G. Groves (eds.), 1967, A GLOSSARY OF OCEAN SCIENCE AND UNDERSEA TECHNOLOGY TERMS: Arlington Compass Publications, Inc., 220 p.

King, C.A.M., 1962, INTRODUCTION TO OCEANOGRAPHY: New York, John Wiley.

Kinsman, B., 1965, WIND WAVES: THEIR GENERATION AND PROPAGATION ON THE OCEAN SURFACE: The Johns Hopkins Univ., 676 p.

Lawrence, L.G., 1967 ELECTRONICS IN OCEANGRAPHY: Indianapolis, Howard W. Sams, 288 p.

McLellan, H.J., 1965, ELEMENTS OF PHYSICAL OCEANOGRAPHY: London, Pergamon, 150 p.

Neumann, G. and Pierson, W., 1966, PRINCIPLES OF PHYSICAL OCEANOGRAPHY: New York Univ., 512 p.

Phillips, O.M., 1966, THE DYNAMICS OF THE UPPER OCEAN: Cambridge, Cambridge Univ. Press.

Pickard, G.L., 1963, DESCRIPTIVE PHYSICAL OCEANOGRAPHY: Oxford, New York Pergamon, 200 p.

Proudman, J., 1952, DYNAMICAL OCEANOGRAPHY: New York, Dover Publications, Inc., 409 p.

Robinson, A.R., (ed.), 1963 WIND-DRIVEN OCEAN CIRCULATION: New York, Blaisdell Pub. Co., 161 p.

Stommel, H., 1960, THE GULF STREAM: Berkeley, University of California Press, 202 p.

Sverdrup, H.U., Johnson, M.W. and Fleming, R.H., 1942, THE OCEANS: THEIR PHYSICS, CHEMISTRY, AND GENERAL BIOLOGY: Englewood Cliffs, N.Y., Prentice-Hall, 1087 p.

Von Arx, W.S., 1962, INTRODUCTION TO PHYSICAL OCEANOGRAPHY: Reading, Addison-Wesley Pub. Co., Inc., 422 p.

Wiegel, R.L., 1964, OCEANOGRAPHICAL ENGINEERING: Englewood Cliffs, N.Y., Prentice-Hall, 532 p.

Yasso, W.E., 1965, OCEANOGRAPHY: New York, Holt, Rinehart and Winston, Inc., 176 p.

Annual Volumes:

Barnes, H. (ed.), 1963-, OCEANOGRAPHY AND MARINE BIOLOGY: London, George Allen and Unwin.

Sears, M. (ed.), 1963-, PROGRESS IN OCEANOGRAPHY: London and New York, Pergamon and Macmillan.

Journals:

Technical - contain original technical articles:

ATMOSPHERIC AND OCEANIC PHYSICS: (Academy of Sciences, USSR)
Translated and Published by: American Geophysical Union
Suite 506, 1145 19th Street, N.W.
Washington, D.C. 20036
(Vol. 1, 1965) QC 880/A1A8

DEEP-SEA RESEARCH: Pergamon Press
Headington Hill Hall, Oxford

(Vol. 1, 1953) GC 1/D3

JOURNAL DU CONSEIL: Cons. Perm, Inter. pour l'Explor. de la Mer
Copenhagen
(Vol. 1,1926)

JOURNAL OF GEOPHYSICAL RESEARCH: American Geophysical Union
Suite 506, 1145 19th Street, N.W.
Washington, D.C. 20036

(Vol. 1, 1949) QC 811/T3

JOURNAL OF MARINE RESEARCH: Sears Foundation for Marine Research
Editor: Yngve H. Olsen
Box 2025, Yale Station
New Haven, Conn. 06250
(Vol. 1, 1937) Gc 1/J6

JOURNAL OF OCEAN TECHNOLOGY: The Marine Technology Society
now: MARINE TECHNOLOGY SOCIETY JOURNAL 1030 15th Street, N.W.
Washington, D.C. 20005
(Vol. 1, 1967)

LIMNOLOGY AND OCEANOGRAPHY: Allen Press
Lawrence, Kansas 66044

OCEANOLOGY: (Academy of Sciences, USSR)
Translated and Published by: American Geophysical Union
Suite 506, 1145 19th Street, N.W.
Washington, D.C. 20036
(Vol. 1, 1960) Engr. Lib./current issues inquire at desk

OCEANOLOGY AND LIMNOLOGY: Omnipress
P.O. Box 395
Haddonfield, New Jersey 08033
(Vol. 1, 1967)

TELLUS: Tellus
P.O. Box 19 111
Stockholm 19, NORWAY
(Vol. 1, 1949) QE 500/T4

Trade - contain professional notes of interest, status of pertinent legislation, editorials, new equipment notes and occasional summary of technical articles:

GEO-MARINE TECHNOLOGY: INTEL, Inc.
739 National Press Bldg.
Washington, D.C.

(Vol. 1, 1964)

INTERNATIONAL MARINE SCIENCE: UNESCO
Office of Oceanography
Unesco, Place de Fontenoy
Paris 7^{e}, FRANCE

(Vol. 1, 1962) Engr. Lib./current issues only on open shelves

OCEAN INDUSTRY Gulf Publishing Company
Box 2608
Houston, Texas 77001

(Vol. 1, 1966)

OCEANOLOGY INTERNATIONAL: Industrial Research Publications Co.
Beverly Shores, Indiana 46301

(Vol. 1, 1966)

THE SEAHORSE. Hydro Products
Division of Oceanographic Engineering Corp.
P.O. Box 2528
San Diego, California 92112

(Vol. 1, 1965)

UNDER-SEA TECHNOLOGY: Compass Publications, Inc.
Suite 1000, 1117 North 19th Street
Arlington, Virginia 22209

(Vol. 1, 1959)

ADDITIONAL BOOKS

Davis, R. A., 1972, PRINCIPLES OF OCEANOGRAPHY: Reading, Mass., Addison-Wesley, 434 p.

Gross, M. G., 1972, OCEANOGRAPHY, A VIEW OF THE EARTH: Englewood Cliffs, N. J., Prentic-Hall, 581 p.

Herring, P. J. and Clarke, M. R. (Eds), 1971, DEEP OCEAN: New York, Pergamon Publication, 320 p.

Duxbury, A. C., 1971, THE EARTH AND ITS OCEANS: Reading Mass., Addition-Wesley, 381 p.

Dietrich, G. and Ulrich, J., 1968, ATLAS ZUR OZEANOGRAPHIE: Biblo. Inst.-Mannheim - Band 7 - Mexers Grosser Physischer Weltatlas, 76 p.

UNIVERSITY OF CALIFORNIA
CE 201A Fall 1970

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OCEAN ENGINEERING

THE ISO-GLOSSARY

A major complaint of engineers confronted by oceanographic literature is the preponderance of maps, profiles, and charts graphing some "iso" quantity. In most cases the root of the iso-quantity term can be guessed by analogy with some more common term. For example, iso-therm (thermometer, thermister, thermastat - all having to do with temperature) must denote something about temperature. "Aha!" the bright student leaps to his feet and exclaims, "isotherm must be the temperature at which ice forms." So much for analogy, as "iso" is the Greek word for equal, not ice.

iso- (Gr. isos: equal)

isoanemone (Gr. Anemos: Wind or inhalation)
Line of equal wind velocities

units: feet/second
miles/hour

isobar (Gr. baros: weight) line connecting values of equal pressure

units: decibar = 1×10^5 dynes/cm^2
(1 atmosphere = 1.0133×10^6 dynes/cm^2)

independent variables: salinity, temperature

isobase (Gr. Basis: A stepping, step, a base, pedestal) line connecting areas of equal uplift or subsidence per given unit of time. Used particularily in glaciated areas with respect to elastic rebound caused by removal of overlying ice sheet.

units: feet, meters

isobath

(Gr. bathos: depth) line connecting points of equal depth, not necessarily sea bottom depths

units: meters, fathoms

isochore

(Gr. chora: space) line representing variation of pressure with temperature at constant volume

units: liters

independent variables: temperature, pressure

isochron

(Gr. chronos: time) line representing equal time, usually geologic

units: years

isocline

(Gr. klinen: to incline) line connecting points on Earth's surface with equal dip of a magnetic needle measured from the horizontal

units: degrees of arc

isentrophe

(Gr. trope: a turning, change) surface of equal entrophy approximately equal to a surface which is adiabatic or $dQ=0$ and since $ds=dQ/T$, is also isentrophic. For practical purposes, $\sigma_\theta = \sigma_T$.

units: see isopycnal (σ_T)

isogonic

(Gr. gonia: angle) line connecting points on Earth's surface with equal angular variations of the magnetic compass needle from true North

units: degrees and minutes of arc

isogram

(Gr. gramma: that which is written) line connecting points of equal value of anything = isopleth, which is more commonly used.

isohaline

(Gr. hals: salt) line of equal salt content or salinity

units: $^{o}/oo$, ppt, parts per thousand

isohyet

(Gr. hyetos: Rain) - Line of equal rainfall

units: inches, centimeters

isogyre (Gr. gyros: ring) circular or near circular line connecting equal values of some parameter. For example, isogyres of the Gulf Stream system may be isopleths of water transport. Isogyre is used generally in oceanographic terminology for circulation parameters where the isopleths are near circular

units: variable

isopleth (Gr. plethos: quantity, number) line connecting points of equal value of anything. Preferred but equal to isogram

isopycnal (Gr. pyknos: thick, dense) line connecting values of equal density (also isopycnic)

units: nominally $\sigma_{S,T,P} = (\rho_{S,T,P} - 1)\ 1000$

more commonly σ_T density at given temperature

independent variables: salinity, temperature, pressure

iso-seismal (Gr. seismos: earthquake) line connecting points of equal seismic activity usually decaying in intensity away from epicenter

units: Richter intensity semi qualitative - log acceleration (cm/sec^2) = intensity/3 - 1/2

isostere (Gr. stereos: solid) line connecting values of equal specific volume

units: cm^3/gm, m^3/ton, or centiliters/ton

independent variables: salinity, temperature, pressure

isotach (Gr. tachos: speed) line connecting values of equal velocity

units: cm/sec, ft/sec

isotherm (Gr. therme: heat) line connecting values of equal temperature

units: °C (degrees centigrade)

independent variables: pressure

University of California
Fall

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C.E. 201A

BAY FIELD TRIP

"Being at sea is like being
in prison, except there is
the additional opportunity
for drowning."

—Samuel Johnson

You are a member of an engineering firm that has been contracted to survey the waters of central San Francisco Bay to recommend possible sites for combined discharges of wastes from some of the existing north San Francisco Bay outfalls. Figure 1 shows the location of the outfalls, and Table 1 lists the characteristics of the waste discharges of interest. Figure 1 and Table 1 are taken from:

Storrs, P. N., R. E. Selleck, and E. A. Pearson, "A Comprehensive Study of San Francisco Bay, 1963-1964," Fourth Annual Report, Univ. of Calif. SERL Report No. 65-1, 1965.

You and the members of your group using the survey ship whose specifications are outlined in Table 2 shall in the time allotted execute a outFall data-gathering trip to supply the information needed for the ~~intake~~ site location study. The equipment listed, with the exception of the water sample bottles, shall be considered part of the ship's equipment and will be on board the ship. However, your group shall be responsible for calibration, maintenance, and operation of the equipment at sea. Any additional equipment and tools deemed necessary to the operation by your group shall be brought to and retrieved from the ship by your group. Water sample bottles shall be provided by each party in numbers necessary for successful cruise operations.

Each individual shall write a formal report which shall include but not limited to the following:

1. Abstract - summary of what's inside including conclusions
2. Purpose of the study
3. Background data and previous work in area
4. Outline of original cruise objectives
5. Personnel on cruise
6. Operational cruise log
7. Equipment used
8. Raw data
9. Corrected data
10. Data interpretation
11. Recommendations on site locations with environmental impact statements for each site
*12. Lessons learned and hindsight cruise plan
13. Recommendations on possible additional surveys
14. References USGS Form

The report will be illustrated with readable and understandable maps, graphs and charts.

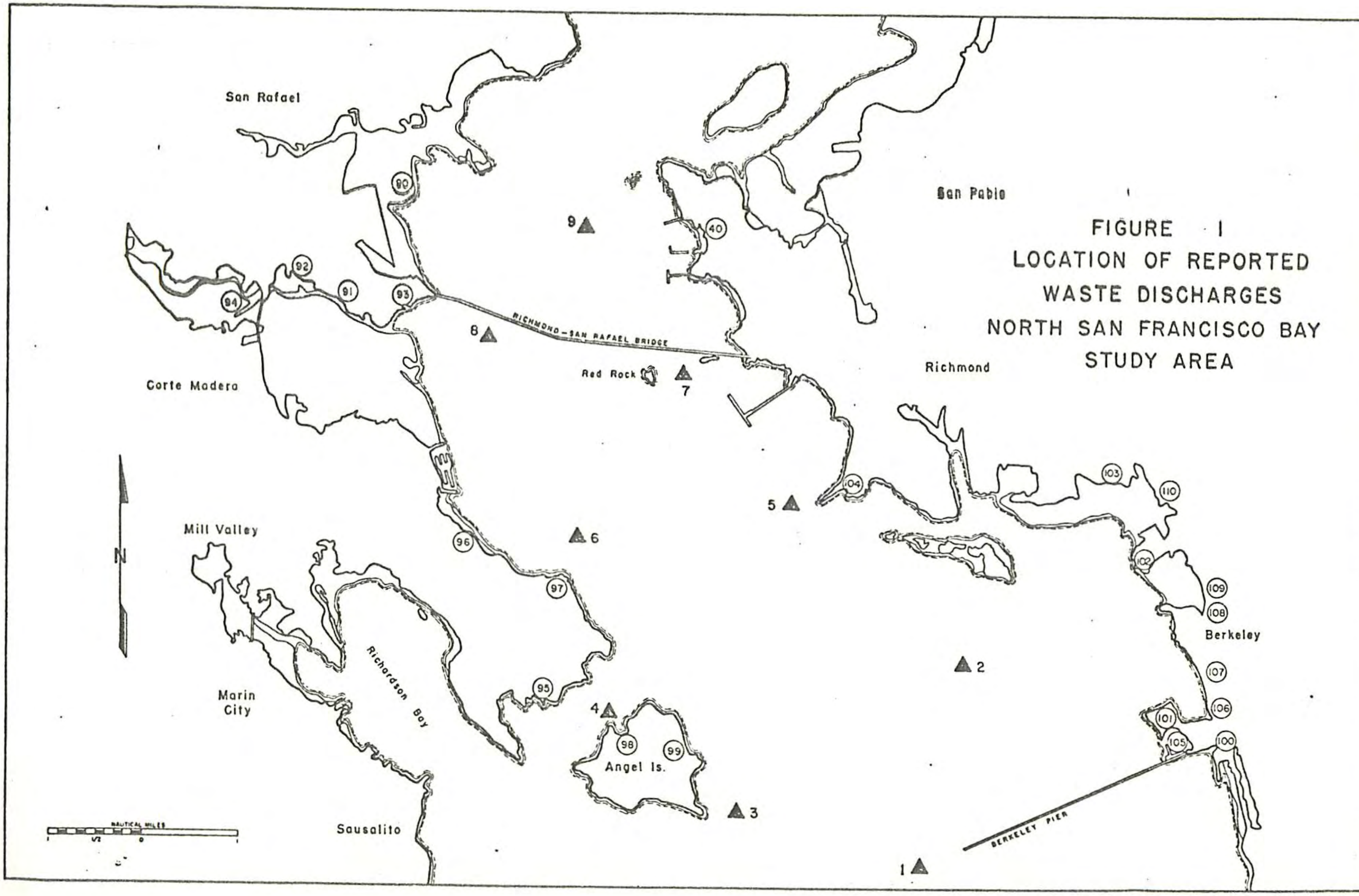


FIGURE 1
LOCATION OF REPORTED
WASTE DISCHARGES
NORTH SAN FRANCISCO BAY
STUDY AREA

TABLE 1

Characteristic	Units	102 Stege Sanitary District			
		Concentration			Load lb/day
		Mean	Min.	Max.	
Flow	mgd	3.20	2.95	4.34	
Temperature	°C	19.3	16.0	23.0	
pH		7.4	7.2	7.7	
Dissolved oxygen	mg/l	4.1	2.6	5.1	110
Dissolved sulfide	mg/l	0.2	0	0.5	5
Total sulfide	mg/l	0.2	0	0.7	6
Settleable solids	ml/l	0.2	< 0.1	0.7	99[a]
Volatile suspended solids	mg/l	69	42	86	1840
Total suspended solids	mg/l	81	48	112	2180
Biochemical oxygen demand	mg/l	109	76	145	2920
Chemical oxygen demand	mg/l	282	220	340	7560
Chloride	mg/l		188	1208	
Phenol	μg/l				
Oil and Grease	mg/l	32	2	42	847
Cadmium	μg/l	10	< 10	20	
Chromium	μg/l	20	< 10	80	1
Copper	μg/l	22	< 10	120	1
Lead	μg/l	14	< 10	40	
Zinc	μg/l	117	< 10	870	3
Dissolved silica	mg/l	16.8	12.2	24.6	452
Phosphate	mg/l	35.9	23.4	54.7	962
NH_3 Nitrogen	mg/l	22.5	15.1	26.7	603
NO_3 Nitrogen	mg/l	0.1	0.1	0.2	3
Total nitrogen	mg/l	29.4	24.3	37.3	787
Coliform MPN/100 ml		3.7e7	6.2e6	> 2.4e8	4.5e15[b]
48 hr TL_m (fresh water)	%	70	56	> 100	
48 hr TL_m (brackish water)	%	86	66	> 100	

[a] Cu ft/day. [b] MPN/day; 4.9e7 =

TABLE I (continued)

MEAN WASTE DISCHARGE CHARACTERISTICS, NORTH SAN FRANCISCO BAY

1963-64

Characteristic	Units	103 Stauffer Chemical Company, Richmond								104 City of Richmond			
		Effluent				Influent							
		Concentration			Load lb/day	Concentration			Load lb/day	Concentration			Load lb/day
		Mean	Min.	Max.		Mean	Min.	Max.		Mean	Min.	Max.	
Flow	mgd	3.10	1.39	4.73		2.94	1.19	4.40		5.95	5.37	7.97	
Temperature	°C	17.1	8.9	23.1		17.2	9.2	22.1		19.1	15.8	23.7	
pH		6.1	4.2	7.9		7.2	5.9	8.0		6.9	5.9	8.8	
Dissolved oxygen	mg/l	8.2	5.2	11.1	211	7.3	5.5	9.3	180	4.6	1.0	6.2	226
Dissolved sulfide	mg/l	0.1	0.1	0.1	3	0.1	0.1	0.1	2				
Total sulfide	mg/l	0.1	0.1	0.1	3	0.1	0.1	0.1	2				
Settleable solids	ml/l	0.2	< 0.1	1	64[a]	0.1	< 0.1	0.2	49[a]	0.6	< 0.1	1.6	527[a]
Volatile suspended solids	mg/l									113	42	176	5620
Total suspended solids	mg/l									148	66	232	7350
Biochemical oxygen demand	mg/l									198	100	430	9960
Chemical oxygen demand	mg/l	675	65	1250	17200	803	254	1350	19100	481	145	1140	24500
Chloride	mg/l		12380	20000			13570	18130			229	590	
Phenol	µg/l									380	110	845	19
Oil and Grease	mg/l									44	28	86	2160
Cadmium	µg/l	10	< 10	20		< 10	< 10	< 10		18	< 5	130	4
Chromium	µg/l	30	< 10	100	1	35	< 10	90		140	< 10	1400	7
Copper	µg/l	23	< 10	140	1	22	< 10	80		23	< 10	110	1
Lead	µg/l	22	< 10	100		12	< 10	30		36	< 10	220	2
Zinc	µg/l	243	< 10	1500	6	18	< 10	60		162	< 10	1400	8
Dissolved silica	mg/l	10.6	2.1	35.0	252	4.8	2.1	9.2	118	20.6	11.9	49.7	1010
Phosphate	mg/l									31.9	13.4	56	1590
NH_3 Nitrogen	mg/l									20.4	12.3	31.8	1000
NO_3 Nitrogen	mg/l									0.4	0	1.0	22
Total nitrogen	mg/l									28.5	16.0	40	1410
Coliform MPN/100 ml										1.3e5	< 4.5e1	> 2.4e8	6.7e13[b]
48 hr TL_m (fresh water)	%	> 100	> 100	> 100						58	24	> 100	
48 hr TL_m (brackish water)	%	> 100	> 100	> 100						70	23	> 100	

[a] Cu ft/day. [b] MPN/day; 4.9e7 = 4.9×10^7.

TABLE 2

SHIP AND EQUIPMENT SPECIFICATIONS

Name of Ship: "Barca I"

Make: Scottie Craft, Miami Beach, Florida

Type: Fast Fiberglass Cruiser

Length Overall: 31 feet

Beam Extreme: 8 feet

Draft: Units down 2 feet
Units up 1 foot

Displacement: 5,000 pounds

Speed: Maximum 35 knots
Economy 25 knots
Lowest 2 knots

Range: Maximum 200 NM

Propulsion: 2 gas Mercruisers inboard and outboard

Power: 2 x 160 HP
Maximum @ 4,000 RPM

Electric Power Available: 12 VDC from 3 x 75 A/h batteries and 2 x 35A alternators

115 VAC from 350 W CD-AC converter

Davit: Stern - 400 pounds maximum load

Dry Storage: 32 cubic feet in drawers and wells

Scientific Equipment:

X ~~Induction salinometer~~: Beckman Model RS5-3 with 250 feet cable: three scales conductivity, temperature, and salinity; battery operated - venier read-out (not available)

✓ Oxygen meter: Martek Model with 100 feet cable: gold oxygen sensor, auxiliary temperature scale

Water sampler: (1) Kemmerer type 5-liter capacity; with slide wire trip messenger - (19) Niskin type 1.9-liter capacity with conventional messenger

Water bottles: Citrate type (bring)

Wire angle meter: Protractor type

~~Current meter:~~ Hydro products Model 502 (not available)

Current Speed

Low range: 0-1 kt. accuracy 3% full scale
High range: 0-5 kts.

Current Direction

353 ± 2° accuracy ± 5°

Temperature

0 - 40°C accuracy ± 3% reading

SOME *DO'S* AND *DON'TS* FOR MARINE OPERATIONS

DO:

(a) plan ahead for possible weather and tide conditions

(b) be pessimistic about equipment reliability

(c) allocate specific jobs to specific people

(d) test gear beforehand

(e) dress warmly in work clothes

(f) secure shipboard end of line going over the side with equipment attached

DON'T:

(a) stand in the bight (open loop) of any line (see Fig. 3)

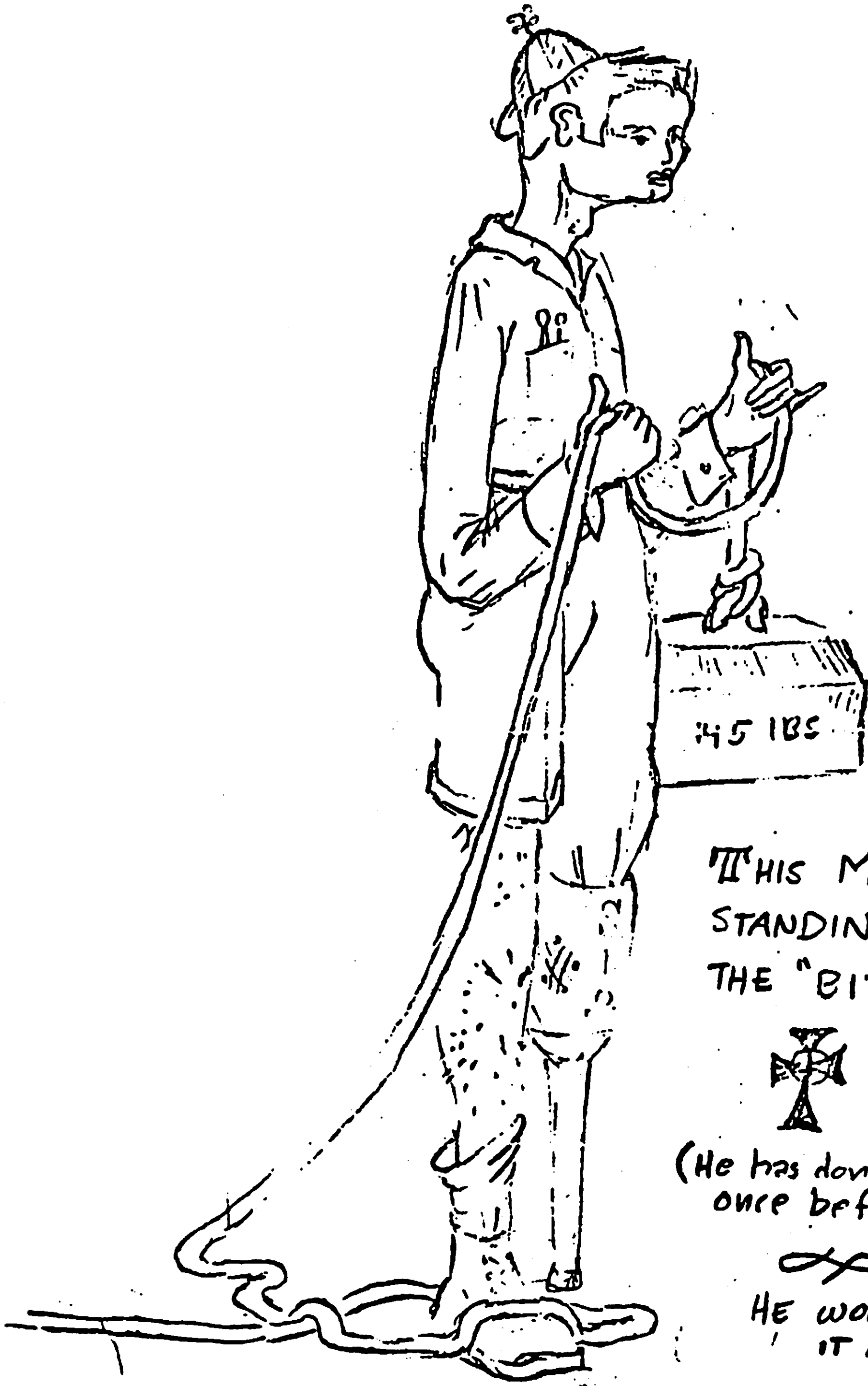
(b) put a strain on electrical cable or use electrical cable as a weight bearing wire (see Fig. 4). Instead, use an independent strain line and tie loosely the electrical line to the strain line.

(c) have too many bosses (see Fig. 5)

(d) If you don't use voice commands, use standardized hand signals (see Fig. 5)

Illustrations courtesy of John Holden, Atlantic Research Lab. (N.O.A.A.)

Figure 3



THIS MAN IS STANDING IN THE "BITE."

(He has done it once before)

HE WONT DO IT AGAIN...

Figure 4

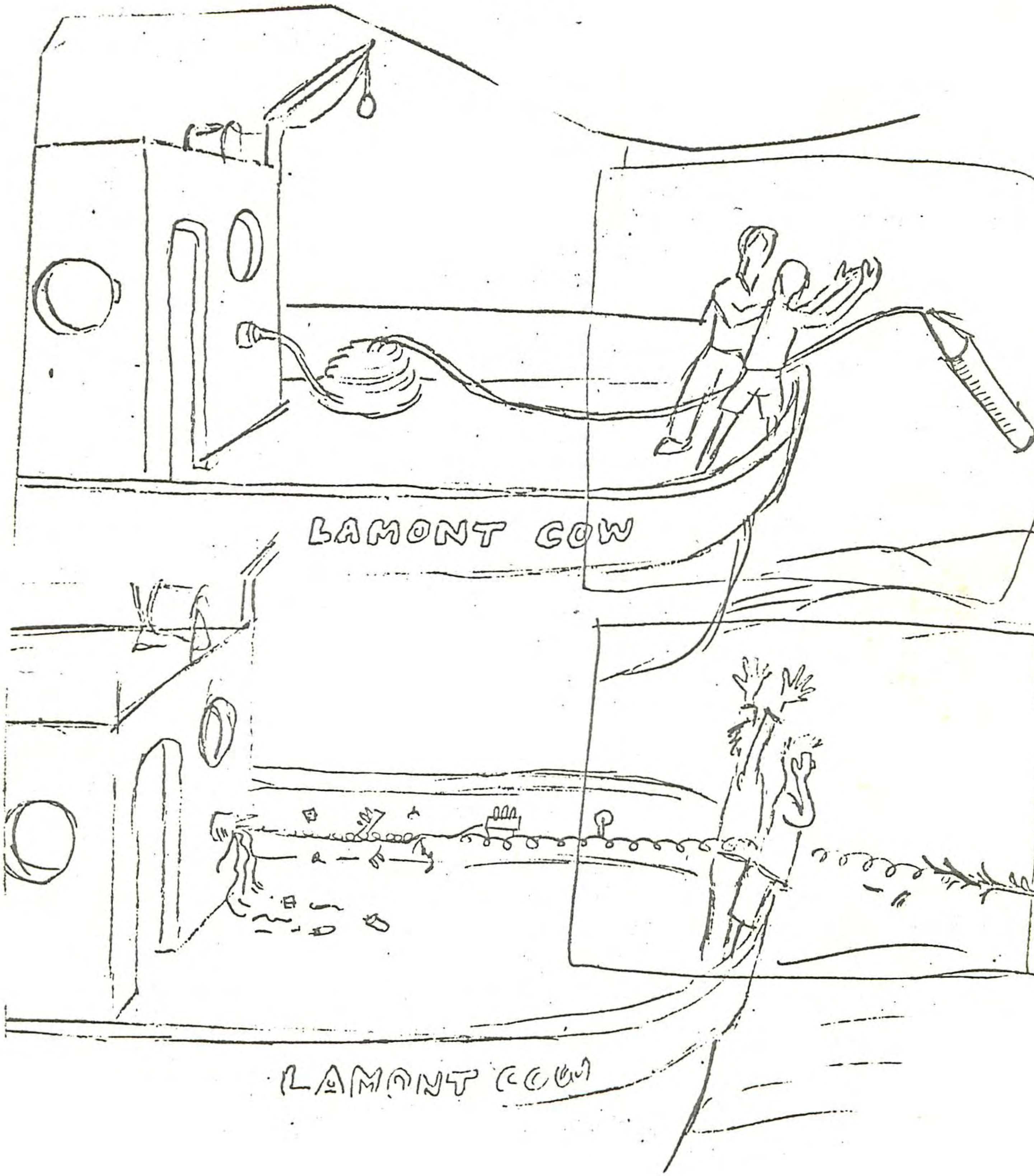
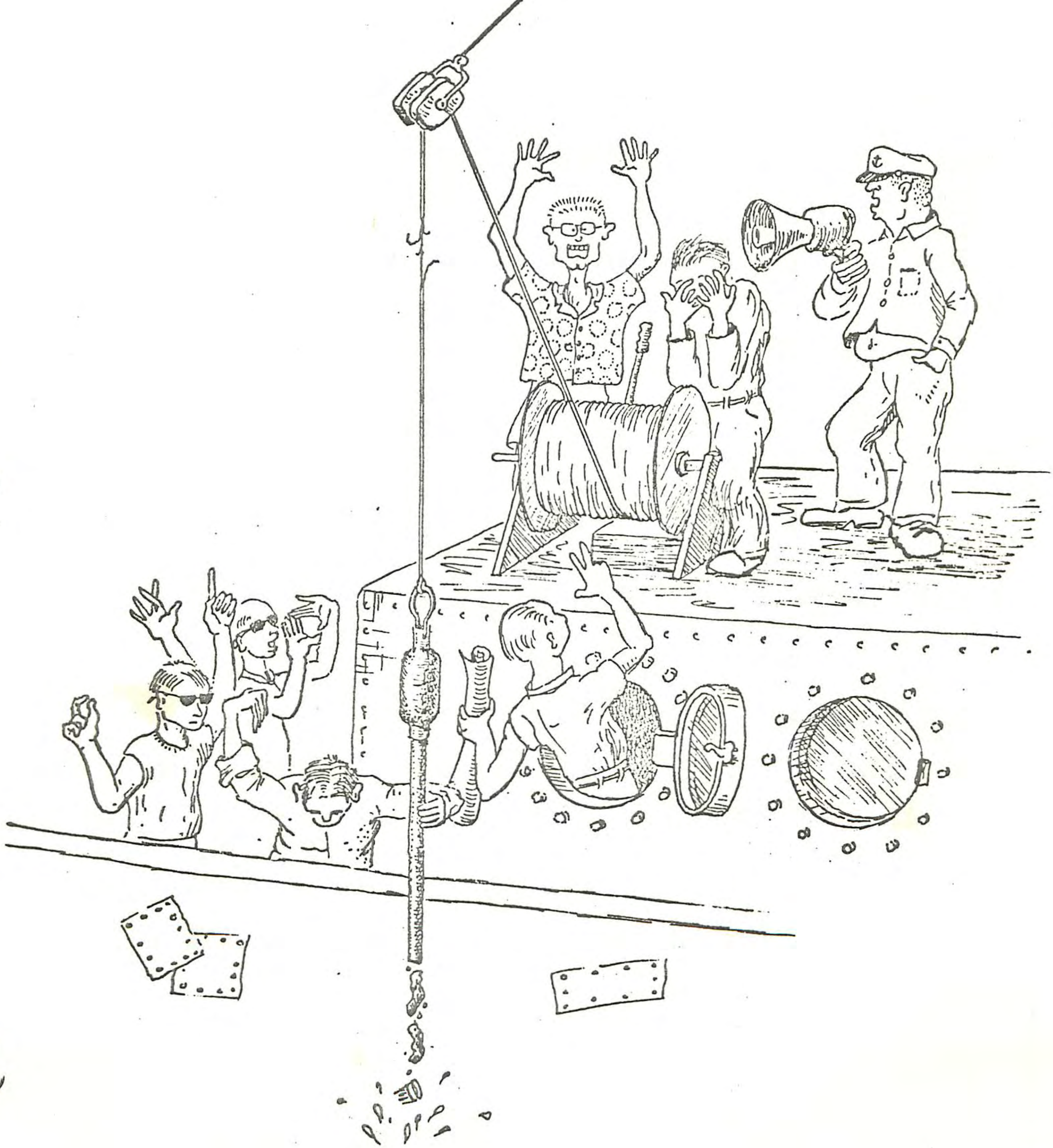


Figure 5



University of California
Fall Quarter

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C.E. 201A

Physical Oceanology

Problems

1. Using a depth sounder with a fixed speed of sound of 800 fathoms per second, what depth interval is recorded for

   (a) 0.5 second sweep
   (b) 1 second sweep
   (c) 2.5 second sweep

2. What is the sign of the correction for the actual depth of water in

   (a) tropical surface waters
   (b) tropical deep waters
   (c) high latitude waters

   Explain in terms of regional temperature and salinity effects on the actual speed of sound.

3. The bottom slope on a fathogram record is 45° on a 1 second sweep where vertically 20 fathoms = 2.2 centimeters and horizontally 5 minutes = 16.2 centimeters. With a ship's speed of 10 knots, what is

   (a) the vertical exaggeration
   (b) the real apparent slope
   (c) the actual slope

4. On a camera station with a sonic pinger attached to the camera and a depth of 1584 fathoms

   (a) what many crossings of the direct return and the bottom bounce return occur before the camera is in the bottom
   (b) illustrate what the record would look like

5. Calculate the radius of the circle of returning echoes for a depth sounder with a frequency of (I) 12 KH_z and (II) 3.5 KH_z at bottom depths of

   (a) 500 fms
   (b) 1000 fms
   (c) 2000 fms
   (d) 5000 fms

6. Calculate the horizontal and vertical components of the coriolis deflection on a water particle at a depth of 300 meters in the Cromwell current at

   (a) 0° and 180° W
   (b) what would be the depth and latitude of the particle if it moved to 140° W

7. Evaluate the equation of state (specific volume) as functions of temperature, salinity, and pressure at standard ocean conditions. Give the integral equivalence of the terms of the specific volume anomaly of Bjerknes.

8. Calculate the depth of the lower Ekman layer for a pressure gradient of 0.1 decibars at 4000 meters (water depth 5000 m) at

   (a) 60° N
   (b) 0°
   (c) 30° S

9. Construct for each 10° of latitude curves of the contour interval of dynamic depth versus geostrophic velocity in steps of 1, 2, 3, 4, 5, 10, 15, 20, 30 centimeters per second.

10. Calculate the Ekman transport for a steady east wind of 5 knots over a 1 minute square at 40° N.

11. From data given on the world average wind speed and orientation chart calculate the depth where D = Z for the surface Ekman layer for:

    (a) 30° W
    (b) 180° W

    at 10° intervals of latitude.

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Instructor: Pat Wilde

FUNDAMENTAL OCEANOGRAPHIC PARAMETERS

In Physical Oceanology the basic measured parameters are (1) three dimensional position, (2) temperature, and (3) salinity or total dissolved solid content of sea water. Other important parameters such as pressure, density, water velocity and sound velocity are at least partially derived from knowledge of the above three principal physical oceanographic parameters.

THREE DIMENSIONAL POSITION:

North-South: Parallels of latitude-degrees, minutes, seconds of arc ″ assuming the only great circle parallel the equator as 0° with the North Pole 90°N and the South Pole 90°S; 1° latitude approximately equals 60 nautical miles so 1' latitude ≅ 1 nautical mile ≅. 6080 feet.

East-West: Meridians of longitude - degrees, minutes, seconds of arc assuming Greenwich, England as 0° or the prime meridian (French charts often use Paris as the prime meridian), meridians East of prime meridians to 180° are east longitude, meridians west of prime meridian to 180° are west longitude.

1 Fathom = 6 Ft = 1.83 meters

Up-Down: Depth in linear units - fathoms, feet, meters (U.S. now only major country that uses fathoms for depth measure) datum sea surface.

It would seem obvious that environmental parameters are useless unless one knows where the data are taken. However, in the open ocean the key word where until recently did not denote the same precision or accuracy that we are accustom to for land positioning. In the oceans the sea surface - atmosphere interface is the primary datum chiefly because the most useful measuring platform is a ship that floats on the interface and the interface is relatively easy to define. I am conscientiously avoiding the term: sea level as the sea surface is not level not only from a geodesic standpoint but because of flucuations due to waves and tides.

However, as a vast majority of oceanic observations are, have been, and will be taken from ships and floating platforms such as buoys, the first positioning problem is to locate that platform on the sea surface.

Surface Position: This easily stated question of where has been the goal of mariners ever since Man took to the sea and at present is not obtainable with the precision and accuracy of land based observations. The basic methods used to determine position at sea are (1) sight bearings by sextant, compass, radar, etc. of previously located fixed objects; (2) celestial, using a sextant, chronometer, and astronomical tables with the fixed stars as the reference; (3) radio frequency, using the time difference between pulsed radio signals from three or more stations with fixed and known locations (see table 1), and (4) satellite, using doppler shift of radio signals from an artificial satellite whose orbit and position at any time in orbit is precisely known.

Location by bearings is the simplest method of positioning but demands an accurate chart in order to plot a moving ship's track. If sight bearings are used the reference object must be visable so that this method is limited to the line of sight of the observer with excellent visability conditions. Radar is simply the radio wave analog of line of sight bearings except radar's usefullness is not limited to good weather or daytime. As the radar mast is higher than the platform from which visual sights are taken, the radar range for navigation is greater than for visual bearings.

representative ship position determining systems

System Type and Name	Manufacturer	Range	Accuracy (rms positions)	Users	Frequency	Signal/ Processing Type*	Stations	Approx. User Cost	Required	Remarks
[CO]NVENTIONAL [R]ADIONAVIGATION										
Short Range Cubic Autotape DM-40	Cubic Corp.	80 km	2-10 m	1	3,000 MHz	P-Ph	2	$60,000-90,000, or lease @ $5,000-9,000/mo.	User buys/ leases system	Line of sight
Hydrodist MRB-2	Tellurometer	80 km	3-30 m	1	3,000 MHz	P-Ph	2			Line of sight
Medium Range Decca Hi-Fix (medium power)	Decca Systems	300 km	5-50 m	Multi	2 MHz	CW-Ph	2	$70,000-100,000 or lease @ $5,000-7,000/mo.	User buys/ leases system	Hi-Fix also usable as two-range system by 1-2 users
Lorac B	Seiscor	500 km	5-100 m	Multi	2 MHz	CW-Ph	4			
Long Range Loran A	Sperry Rand, ITT, others	1,200 km	1.5-8 km	Multi	2 MHz	P-Pt	3	$2,000 (Receiver)	Govt. installed shore stations; user acquires receiver	
Loran C		1,800 km	15-400 m	Multi	100 kHz	P-Pt	3	$30,000 (Automatic system)		Limited coverage,
Global Omega	ITT, Tracor, Nortronics	900 km	800-3000 m	Multi	10 kHz	P-Ph	3	Predict Receiver $3-100,000		system to be operational by 1972
POSITION FIXING BY SATELLITE										
AN/SRN-9 receiver + choice of computer	ITT	Global	Single-fix 60-120 m (average fix interval 90 minutes)	Multi	150 and 400 MHz	Doppler	3 or more satellites	$70,000-135,000	Govt. operated satellites, user buys receiver and computer	Includes continuous navigation output
702 receiver + choice of c[omp]uter	Magnavox									
U[...] Geo Navigator	Honeywell									
ACOUSTIC SYSTEMS										
Doppler Sonar MRQ-2015 Doppler sonar	Marquardt	Water depth (alt. above bottom less than 200 m)	0.2-1% of distance traveled	1	350-400 kHz	Doppler	none	$50,000-80,000	User buys system	Relative position only
Kollsman marine doppler docking and navigation system	Kollsman									
Scanning Sonar Scanar II-F sonar	Honeywell	1,200 m	1-10% of range, 1.5-5 deg. bearing	1	100 and 300 kHz	P-Pt & AM		$7,000	User buys system	Relative position only
Sea-Imp marker locator	Honeywell	1,000 to 2,000 m	10% of water	1	50, 175 kHz	P-Pt & AM	1+	$10,000	Sonar and one or more bottom pingers	Relative position only
Model 500 CTFM sonar	Straza	1,800 m	2-10% of range, 2 deg bearing	1	87-72 kHz, 40-55 kHz	CTFM	1+	$30,000	Sonar, pinger optional	Relative position only
Base-Station, Multi-mode RS-4 acoustic position	Honeywell	5-10 km	0.2-1% of range or water depth	1	3.5-24 kHz	P-Pt	1 (up to 5)	$100,000-200,000	Shipboard system + bottom units	can function as range-range, bearing-bearing, or bearing system
Base-Station Ran[ge-]Range SU[...]onic underwa[ter nav]igation syste[m]	Alpine Geophysical Associates	10 km	2-30 m	1+	11-16 kHz	P-Pt	3	$30,000	Shipboard system + three transponders	Requires addition of computer for x-y output
[B]ase-Station [B]earing-Bearing [B]S-3A acoustic [p]osition indicator	Honeywell	20% of water depth	0.2-0.5% of water depth	1	40-60 kHz	CW-Pt and Ph	1	$85,000	Shipboard system + 1 beacon	Site reacquisition and lower precision operation provided at longer ranges

* Signal Type and Processing: P-Pulse, CW-Continuous Wave, CTFM-Continuous Transmission Frequency Modulated

Celestial fixes may be taken anywhere in the world but are limited practically by weather conditions mainly cloud cover as the stars or other heavenly bodies must be seen. In perennially foggy areas of the world the lack of good sky visability seriously limits the use of celestial positioning. Also good star fixes can be obtained only twice a day at dawn and dusk when both sufficient stars and the horizon (used as a reference plane) may be seen together. At best a star fix gives the position to five nautical miles twice a day. The position of a moving ship for any other time must be obtained by dead reckoning (DR) which is plotting forward or backwards in time from a fix assuming the ship's course and speed. Auxiliary celestial fixes of the sun, moon, and the planets help limit the inherent errors of the DR plot, such as variable ship's speed and offset of the apparent course by waves and currents. For ships moving at a reasonably steady speed and a constant course DR positioning is generally adequate. However, for oceanographic ships steering various courses and speed between star fixes and often drifting on station for many hours, plotting the DR track required a skill of the highest order and takes not only a dedicated navigator obtaining excellent fixes but one well conversant in the vagaries of winds and currents as well as the reliability of the ship's engines to maintain a given speed.

Positioning by radio frequency has the advantage of being operative in cloudy weather but has the disadvantage of being a near shore method depending on an established netwerd of shore stations. Any non-line of sight radio method depending on the bouncing of radio waves off the ionosphere which unfortunately does not maintain a fixed height with respect to the earth and add a definite uncertainity to locations obtained particularily at night.

Satellite navigation offers the best hope thus far for obtaining positions anywhere in the world regardless of weather condition. The number of fixes available are a function of the number of navigational satellites passing over the ship per unit time. There usually are four usable satellites each of which orbits the earth in approximately 90 minutes. Under optimum conditions with the satellites passing the ship at even time intervals, this means a fix every 20 minutes which is an obvious improvement over two fixes per day by celestial means. However the satellites are not all in optimal orbits so at present the time between fixes varies.

Each satellite transmits two carrier frequencies, at 150 MH_z and 400 MH_z. Two minute navigational messages are encoded onto both signals by phase modulation. The message consists of two parts: (1) A fixed part (see Fig 1) which defines the smooth ideal orbit and (2) A variable message which gives corrections to the orbit. This combination defines the position of the satellite for eight two minute time intervals. The variable portion changes each two minutes with the addition of the latest fix and the deletion of the oldest position. The navigational message is periodically updated from injection stations who with the aid of computers and fixed tracting stations monitor the orbit of the satellite and transmit to the satellite memory revised orbital parameters.

Thus the receiving station obtains from the satellite, (1) two precise frequencies, (2) the location of the satellite, and (3) time as each navigational message lasts exactly two minutes and begins and ends at the start of each even minute.

To obtain the position of the receiving station, its relative position with respect to the satellite must be known. This is done by measuring the doppler shift of the signals from the satellite. The frequency received (Fr) from the satellite consists of the

FIGURE 1

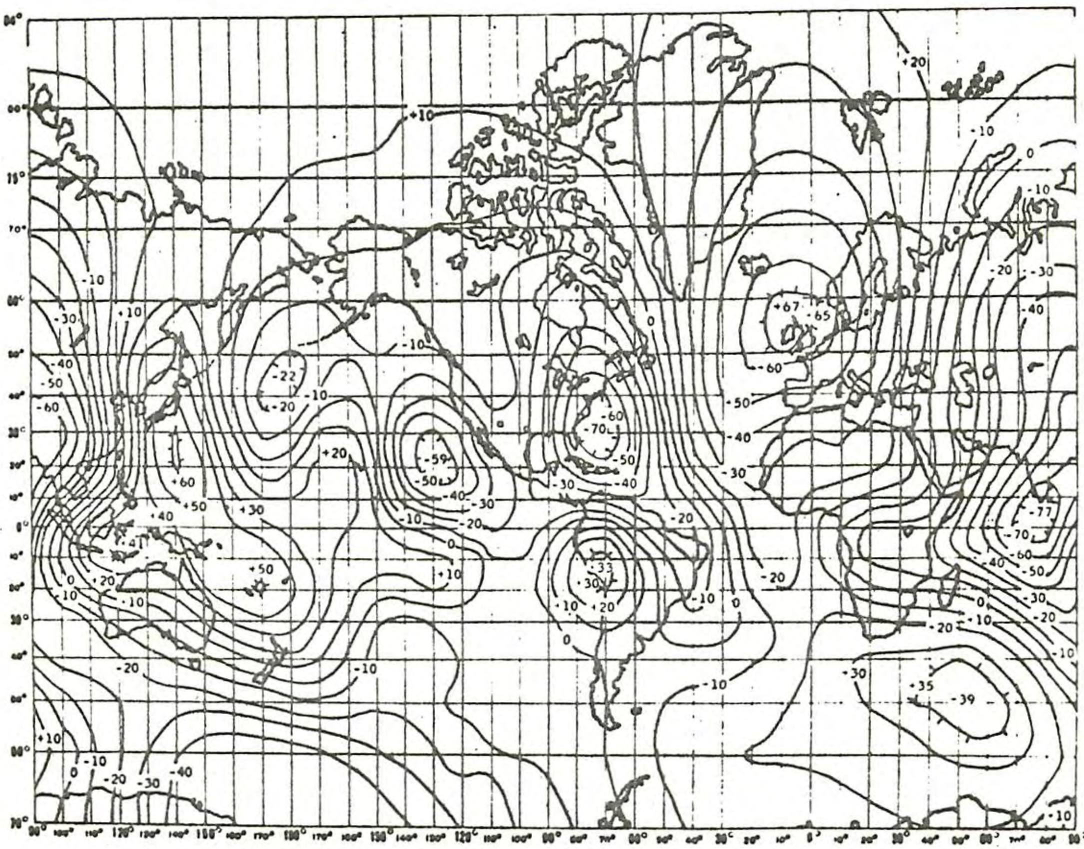


Fig. 6—Contour plot of mean sea level deviations from a reference ellipsoid, in meters, as defined by the gravity model in operational use since January 1966.

WORD NUMBER	SYMBOL	MEANING	UNITS
56	t_p	TIME OF PERIGEE	MIN
62	$\dot{M}$	RATE OF CHANGE OF MEAN ANOMALY	DEG/MIN
68	φ	ARGUMENT OF PERIGEE AT t_p	DEG
74	$\lvert\dot{\varphi}\rvert$	RATE OF CHANGE OF ARGUMENT OF PERIGEE (ABSOLUTE VALUE)	DEG/MIN
80	ϵ	ECCENTRICITY OF ORBIT	—
86	A_e	SEMI-MAJOR AXIS OF ELLIPSE	Km
92	Ω_N	RIGHT ASCENSION ASCENDING NODE AT t_p	DEG
98	$\dot{\Omega}$	RATE OF CHANGE OF Ω_N	DEG/MIN
104	$\cos\Psi$	COSINE OF ORBIT INCLINATION	—
110	Ω_G	RIGHT ASCENSION GREENWICH	DEG
128	$\sin\Psi$	SINE OF ORBIT INCLINATION	—

Fig. 7—Fixed portion of satellite navigation message.

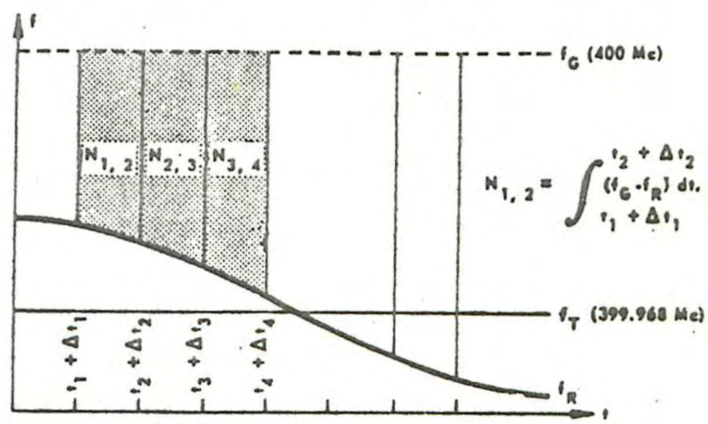


Fig. 8—Implementation of integrated doppler count.

From Stansell (1968, p.232,233)

transmitted frequency (F_t) plus the Doppler shift due to the relative motion of the receiving station and the satellite. The receiving station has a 400 MH_z (F_G) crystal reference frequency oscillator to which the satellite signal is compared. The integrated Doppler count (N) is the count of (F_G-F_t) or ΔF plus the number of Doppler frequency cycles received during the two minute message. A geometric representation of the Doppler count is shown in Fig. 1.

The following derivation is modified from Stansell (1968). As

$$N_{12} = \int_{t_1+\Delta t_1}^{t_2+\Delta t_2} (F_G - F_R)\, dt \quad \ldots\ldots\ldots\ldots (1)$$

$t+\Delta t$ = time of receipt of the message with (t) the time of transmittal and Δt the time delay. As shown on Fig. 2 the slant range distance (s) = the delay time (Δt) times the speed of light (c) or

$$\Delta t = s/c \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Rearranging (1)

$$N_{12} = \int_{t_1+\Delta t_1}^{t_2+\Delta t_2} F_G dt - \int_{t_1+\Delta t_1}^{t_2-\Delta t_2} F_R dt \quad \ldots . \ (3)$$

The first term, as F_G is constant, is easily solved. However, in the second term F_R is variable. But as the number of cycles transmitted must equal the number of cycles received

$$\int_{t_1+\Delta t_1}^{t_2+\Delta t_2} F_R dt \equiv \int_{t_1}^{t_2} F_T dt \quad \ldots\ldots\ldots\ldots (4)$$

or

$$N_{12} = F_G\, [(t_2-t_1)+(\Delta t_2-\Delta t_1)] - F_t\, (t_2-t_1) \quad \ldots \ (5)$$

or

$$N_{12} = (F_G-F_T)(t_2-t_1) + F_G(\Delta t_2-\Delta t_1) \quad \ldots\ldots \ (6)$$

F_G-F_t is assumed constant during a pass and (t_2-t_1) = 120 seconds = ΔT as the wavelength λ

$$\lambda_G \equiv c/F_G \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (7)$$

Eq (6) may be written

$$N_{12} = \Delta F \Delta T + (1/\lambda_G)(s_2-s_1) \quad \ldots\ldots\ldots\ldots \ (8)$$

and solving for Δs_{12}:

$$\Delta s_{12} = (s_2 - s_1) = \lambda_G N_{12} - \lambda_G \Delta F \Delta T \quad . \; . \; . \; . \; . \; . \; (9)$$

The actual position is calculated by a computer at the receiving station by a best fit solution considering (1) the estimated position, (2) the calculated slant range distance, and (3) the transmitted satellite position. Such interations take only a few minutes on small digital computers like the PDP-8.

Sources of Errors in Satellite Fixes

For oceanographic work the major errors are due to the ships motion. Figures 2, 3, and 4 show the fix errors due to such uncertainties. As seen in the graphs such errors are almost negligible when compared to the 5 mile error of a star fix.

FIGURE 2

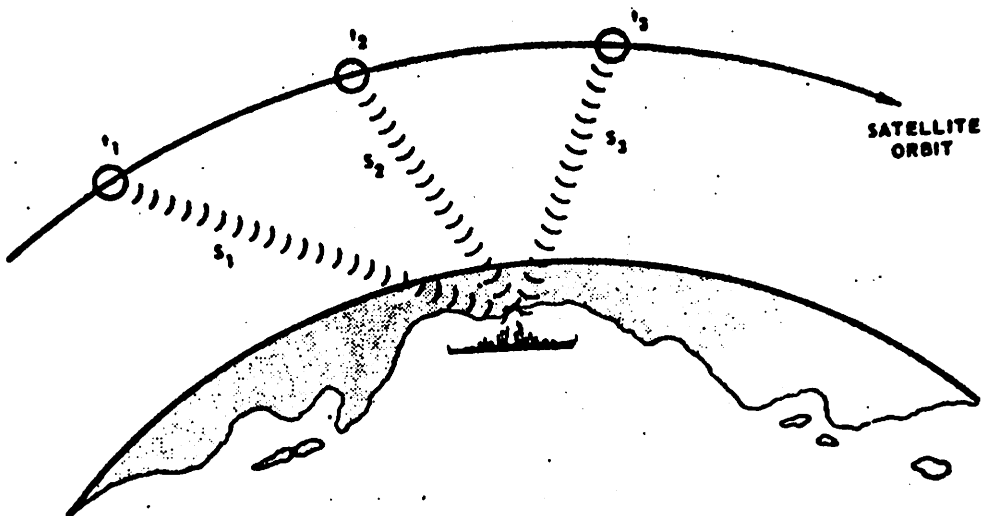


Fig. 9—Geometry of a satellite pass.

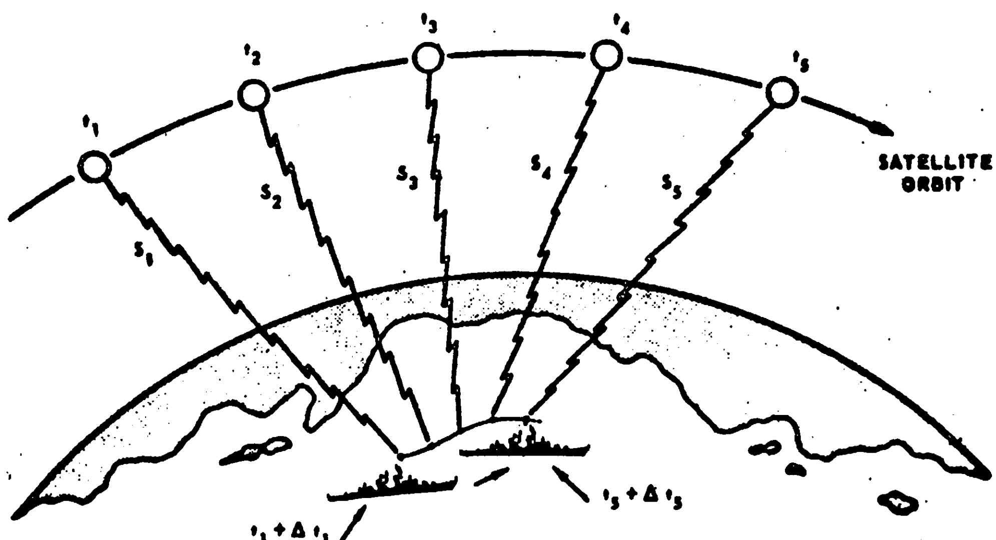


Fig. 10—When underway, ship's motion must be known so that measured slant range differences to the satellite will be geometrically meaningful.

From Stansell (1968, p.234,236)

FIGURE 3

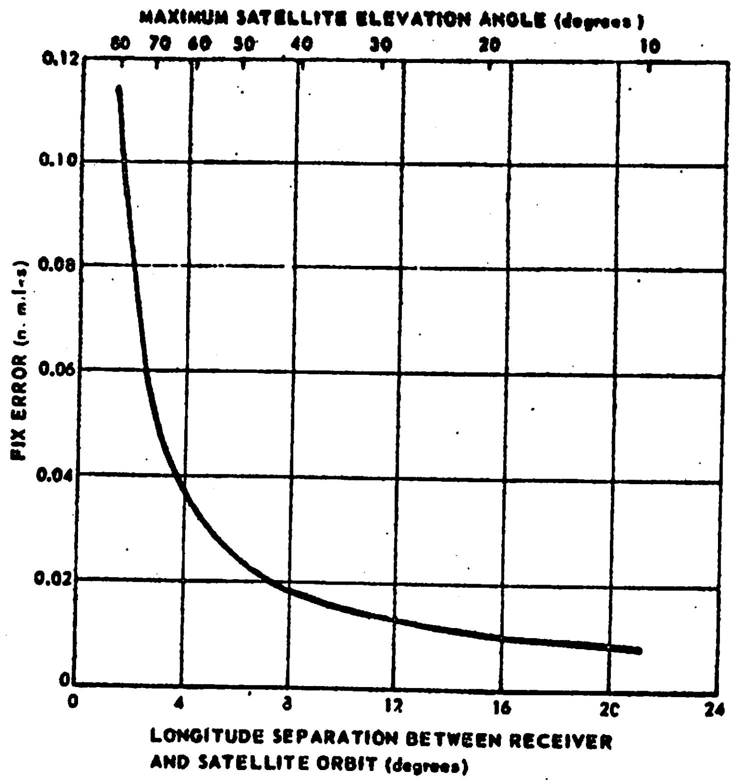


Fig. 11—Fix error resulting from each 100 feet of antenna height error.

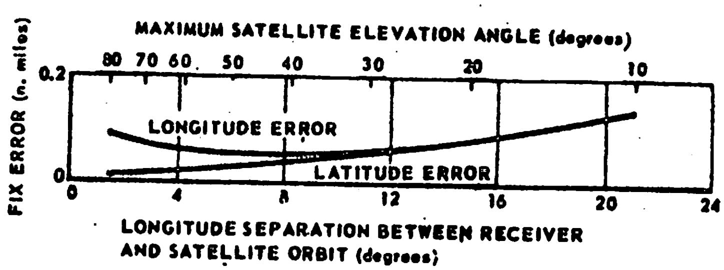


Fig. 12—Fix error resulting from each knot of velocity east error.

From Stansell (1968, p. 237,238)

FIGURE 4

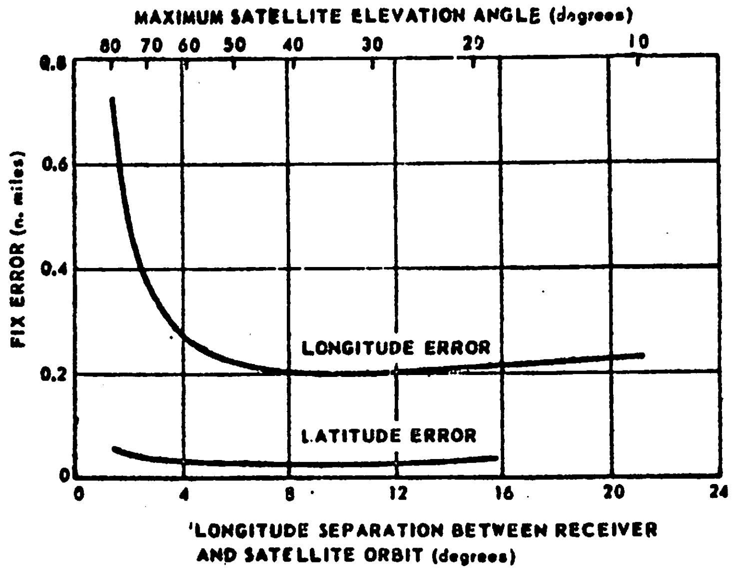


Fig. 13—Fix error resulting from each knot of velocity north error.

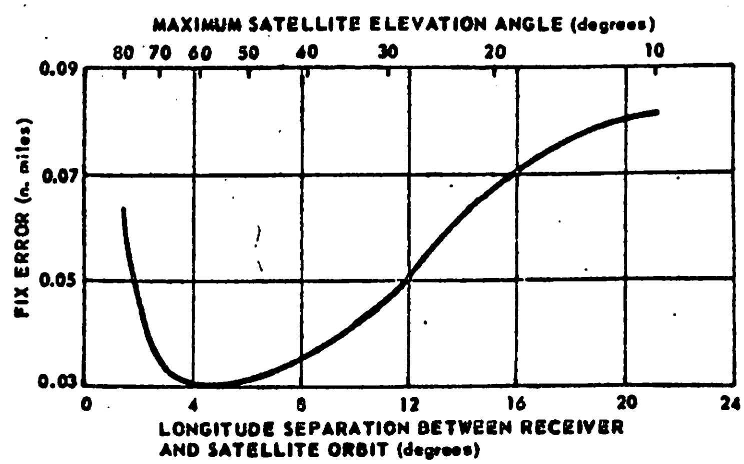


Fig. 14—Average fix error resulting from random errors of 60 ft. RMS in specifying ship's position at two-minute time marks.

From Stansell (1968, P. 238)

anywhere in the world regardless of weather conditions. The number of fixes available are a function of the number of navigational satellites passing over the ship per unit time. At present (Sept 1968) there are four usable satellites each of which orbits the earth in approximately 90 minutes. Under optimum conditions with the satellites passing the ship at even time intervals, this means a fix every 20 minutes which is an obvious improvement over two fixes per day by celestial means.

Depth

The oceanographer is interested also in the variation in environmental parameters in the water column as well as along the sea water surface thus he must know not only the actual depth to the sea floor but the depth at which a particular parameter is measured. It is a strange comment that until the advent of satellite navigation the oceanographer had a better knowledge of the depth parameter than he did of his position at sea on the earth's surface. At sea depth is measured by (1) actual linear measure ment such as by a weighted wire called a lead line; (2) the pressure effect (a function of depth) on the compressibility of mercury in a themometer at a depth which gives an apparent temperature anomaly with respect to a themometer at the same depth but protected against pressure effects; and (3) sonic devices which record the time a sound pulse travels in sea water between a sound source and a receiver.

The classic method of measuring the length of wire needed to touch the bottom was the chief source of depth information until the 1920's. The practical depth unit the fathom owes its length (6 feet) to the fact that the old way of measuring the length of the lead line was by taking increment equal to the length of a man's outstretched arms (about 6 feet) as the line was pulled in. Thus the number of times the leadman stretched his arms was the depth in fathoms. The major error associated with lead lining is that with great depth or strong currents or both the weight the the end of the line can not hold the wire in a true vertical position as does a plump bob, such that the length of wire out is not equal to the actual depth.

To correct for a non-vertical wire angle the pressure effect on apparent temperature is used to determine the depth of any instrument paclage. This effect is measured by a calibrated pair of reversing themometers attached to the instrument (details on the reversing the memeters are given in the section on temperature). The depth is given by

$$Z = \frac{tu - tw}{Q\rho m}$$

Where:

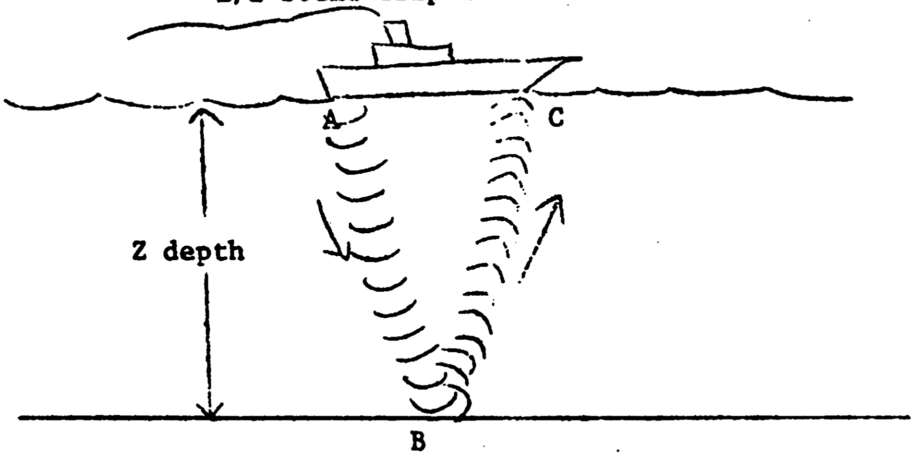
Z = depth in meters
tu = temperature of the unprotected themometer in ^{o}C
tw = temperature of the protected themometer in ^{o}C
Q = pressure constant for calibrated pair of themometers, about o.o1^{o}C/meter
ρm = mean specific gravity of sea water

This method is still used in hydrographic casts with Nansen water sampling bottles.

Lead lining and the use of reversing the thermometer pairs has been replaced by sonic methods in which the depth is determined by precise timing of sound pulses traveling from a sound source (transducer or transponder) to a recorder.

With the sound source and recorder in the same horizontal plane the water depth is determined by timing the sound echo reflected off the bottom or:

$$\frac{\text{Speed of sound in sea water}}{\text{1/2 round trip travel time}} = \text{Bottom depth or Depth to relecting layer}$$



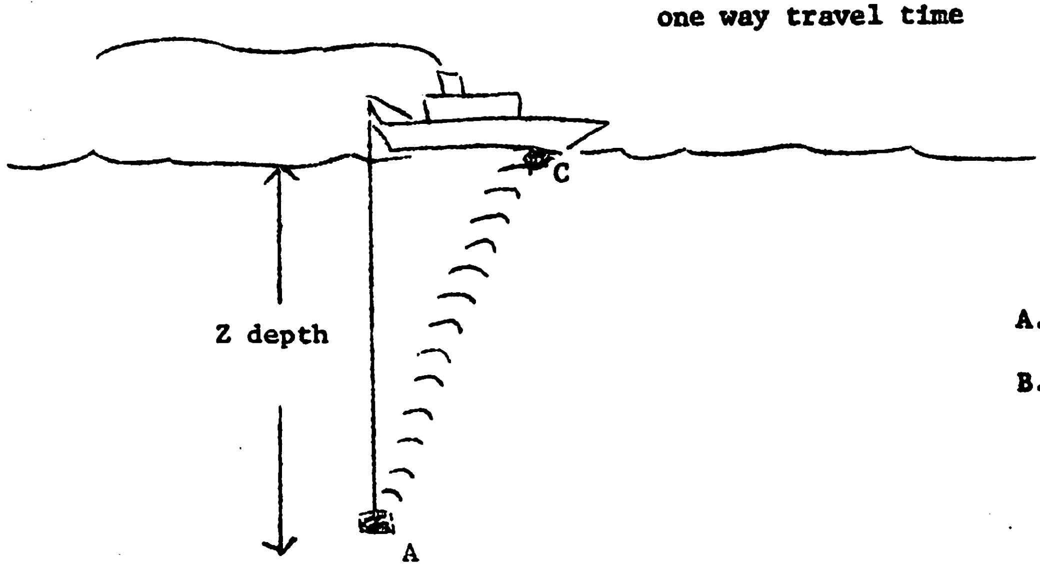
A. sound source

B. Reflecting layer

C. Recorder

Instrument depth determinations in the water column can be made with sonic devices called "pingers" which are simply a sound source attached to the instrument. In this case the sound source and recorder are separate so that the arrival time is a function of the direct path rather than the reflected path:

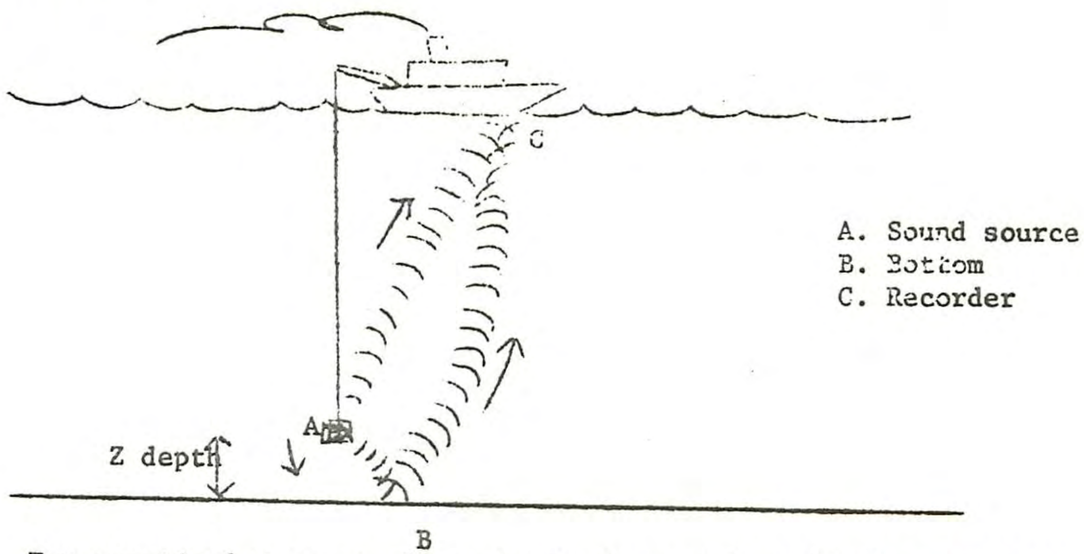
$$\text{Z Depth of Pinger} = \frac{\text{Speed of Sound in Sea Water}}{\text{one way travel time}}$$



A. Sound source

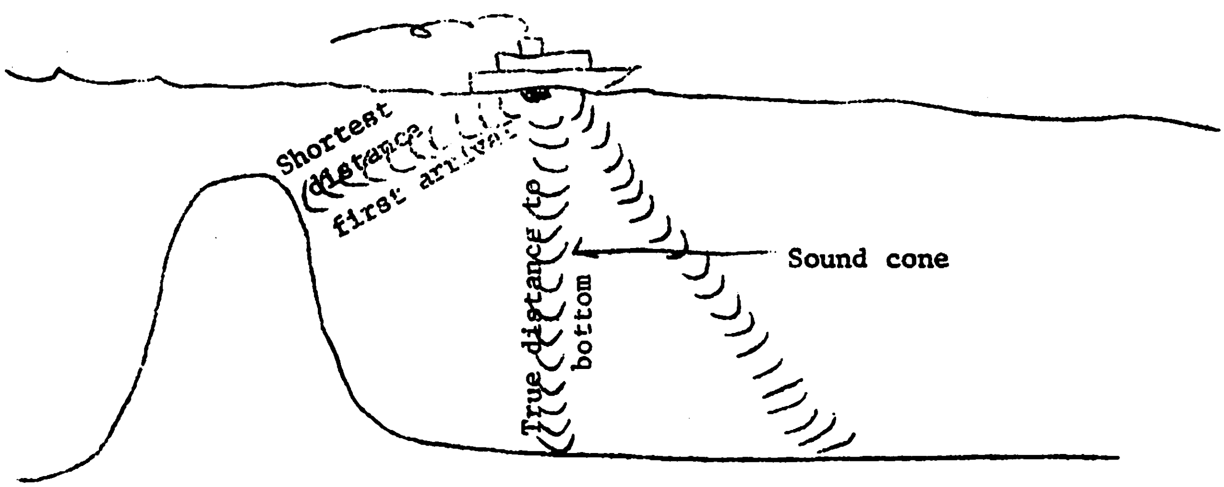
B. Recorder

The acoustic pinger also gives the depth of the pinger instrument package with respect to the bottom. This information is needed for camera stations, etc. Where it is imperative to put the instrument package near the bottom but not on the bottom which potentially could damage the instrument. In this method both (1) the direct path to the ship and (2) the reflected path from the pinger to the bottom to the ship are recorded and the time lag between these two paths determines the distance between the pinger and the bottom

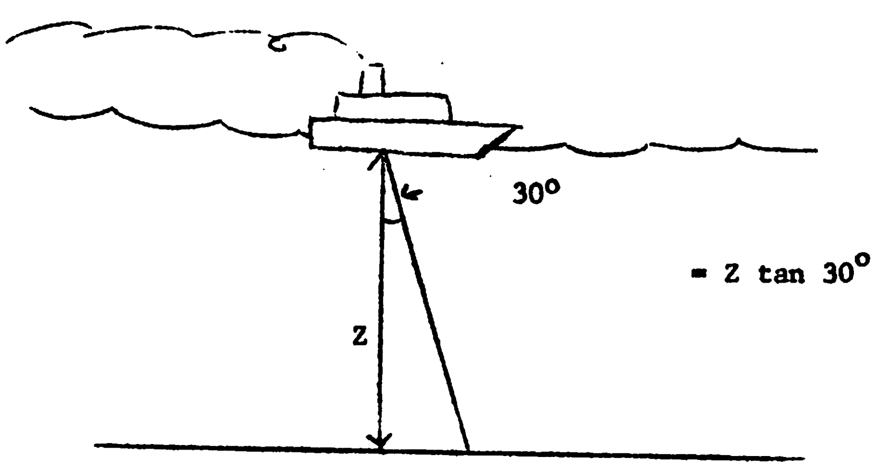


For practical reasons all sonic depth recorders (Fathometer is a trade name of the Raytheon Division of Sylvania Electronics) are calibrated for a constant speed of sound in sea water. Machines recording in fathoms assume a speed of 800 fathoms per second or 1463 meters per second. Machines recording in meters assume a speed of 1500 meters per second or 820 fathoms per second. Corrections for variations in the speed of sound in the water column can be made by use of Matthews' tables (Matthews, 1939, 1948), although these tables are far from being completely reliable or exhaustive in a real coverage. Thus depth by sonic means are precise but their accuracy is limited by the assumption of a constant sound velocity in sea water. Most sonic depths are reported uncorrected so that mariners with similar equipment can read the same value.

Another reduction in accuracy inherent in most sonic depth sounders is the fact that the sound waves leave the transducer as a sound cone with a half-angle of approximately 30° for the 12 kilo hertz frequency normally used. As the first arrival is plotted as the depth obviously in areas of rugged bottom relief the first arrival may be a return echo from a feature not directly below the ship.



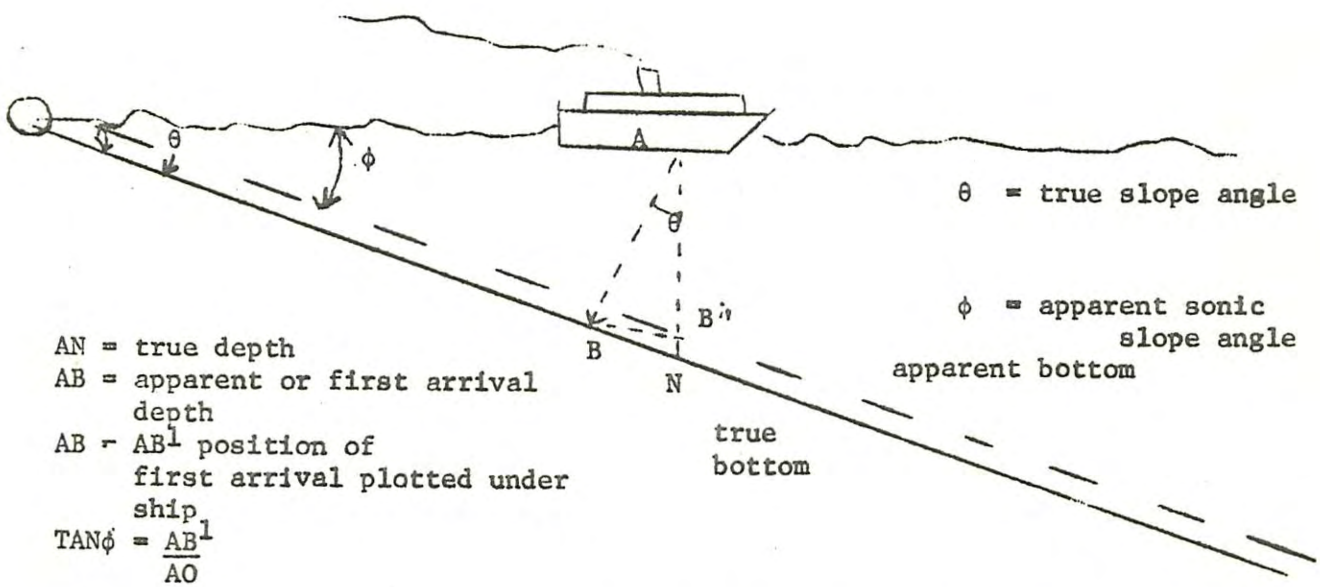
Accordingly the two dimensional record produced by the sonic depth sounder shows a composite picture of the shallowest depths of three dimensional bottom features within the sound cone whose area of investigation increases with depth.



Area seen by recorder = $\pi(.577Z)^2$

taking the average depth of the oceans as 12,000 feet, this means a coverage of about 150 x 10^6 square feet (5.5 square statute miles) or a circle of error of a little more than a statute mile in diameter in which the first arrival depth actually is located. Krause (1962) presents sound cone error corrections for numerous geometrical bottom shapes; although in practice because of the fundamental position error of at least five miles (pre-navigational satellite) such corrections rarely are made.

Also because of the sound cone the first arrival which is usually taken as the bottom return may not be directly under the ship. In a simple case shown below this could lead to an error in the slope measurement.



θ = true slope angle

φ = apparent sonic slope angle

AN = true depth

AB = apparent or first arrival depth

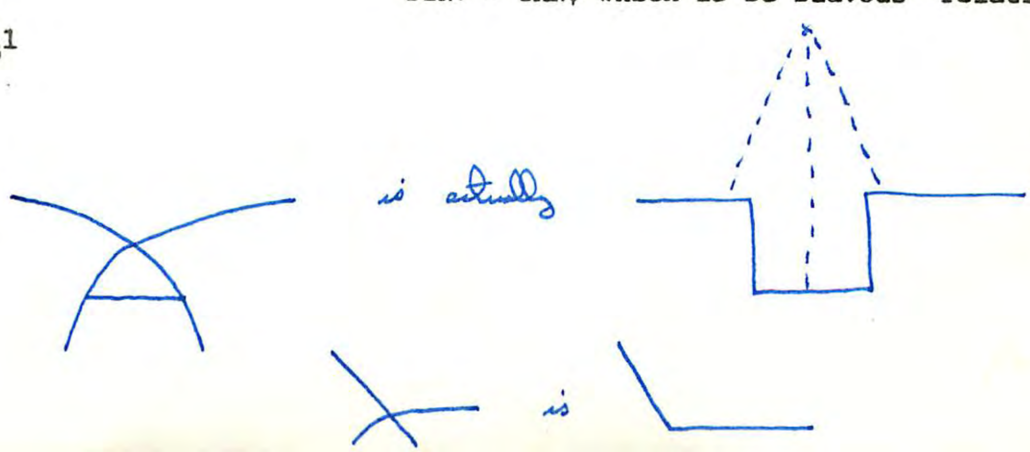
AB - AB^1 position of first arrival plotted under ship

$\tan\phi = \frac{AB^1}{AO}$

$\sin\theta = \frac{AB}{AO}$

as $AB = AB^1$

$\sin\theta = \tan\phi$ which is De Blavous' relationship



Fixed uncorrected sound speeds { 800 Fathoms/sec. or 1500 Meters/sec.

REFERENCES - Three Dimensional Position

Frequency Ranging

Bigelow, H. W., 1964, Electronic Positioning Systems: Under Sea Tech, V. 5, No. 4, p. 24-28.

Satellite Navigation

Freitag, R. F., 1961, Project Travsit Navigation: U.S. Naval Inst. Proceed., V. 87, No. 5, p. 76-83.

Winters, P. R. , 1967, A Navigation Geodetic System by Satellite: U.S. Naval Inst. Proceed., V. 93, No. 6, p. 131-134.

Celestial Navigation

Hill, J.C. and others, 1958, Dutton's Navigation and Plotting: Annapolic, U. S. Naval Inst., 771 p.

Depth

Matthews, D. J., 1939, Tables of the velocity of sound in pure water and sea water for use in echo-sounding and sound ranging: London, Admiralty Hydrographic Office Pub. 282, 52 p. Supplement 1948, Echo Sounding, Errors, adjustments and reporting, 10 p.

Krause, D. C., 1962, Intepretation of echo-sounding profiles, Internat. Hydrographic Rev., v. 39, p. 65-123. } Best One

General Interpretation of position Information

Chase, T. E., 1968, S a Floor Topography of the Central Eastern Pacific Ocean: U.S. Bureau of Commercial Fisheries Circular 291, 33 p.,

ADDENDA

STANSELL, T.A., 1971, EXTENDED APPLICATION OF THE TRANSIT NAVIGATIONAL SATELLITE SYSTEM: OFFSHORE TECHNOLOGY CONF. 1971, p I-707 - I-726.

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TEMPERATURE: °DEGREES CENTIGRADE (CELSIUS), FAHRENHEIT

Temperature can be measured by (1) substances with a linear co-efficient of thermal expansion: mercury thermometer, (2) a temperature sensitive resistor: a resistance thermometer or thermistor, (3) a crystal whose oscillation frequency changes with temperature: quartz thermometer and (4) by infra-red emmissions: radiation thermometer or bolometer.

The classic method of taking the temperature of sea water is with a mercury thermometer. For spot measurements of temperature in the water column a special pair of thermometers called reversing thermometers are used. Reversing thermometers are constructed so that when they are turned over the mercury column breaks and the temperature reading just before the thermometers were reversed is preserved until the thermometers can be read at the surface. Such thermometers (see Fig. 1) are still uilized in hydrographic casts attached to nansen or other water sampling bottles that turn over when triggered.

However continuous temperature data on the surface layer and to a depth of 300 meters (about 1000 feet) come from the use of the Bathythermograph or BT, which permit underway data collection not possible with simple mercury thermometers. The Bathythermograph (Spilhaus, 1938) consists of a towed instrument package in which a bourdon thermal element moves a needle over a smoked or bold plated glass slide at right angles to a pressure sensitive spring calibrated for depth. The accuracy is about 1% for depth, and ±0.2°C for temperature. As the recording element, the glass slide, is scribed in place as the instrument package or fish moves up or down in the water column, the fish had to be recovered back on board the ship to retrieve the data. The conventional BT now is being surplanted by the expendable BT or the XBT in which the fish is simply a weight whose rate of fall in water is predetermined, carrying a thermistor connected to the ship by a thin disposable multistranded wire. A recorder on the ship is geared to advance the paper at the rate of all of the probe. The temperature is recorded as the change of resistance of the thermistor. The disposable parts of the XBT cost about $20.00 (1966) for the Birnican Corporation model.

Radiation thermometers (see Table II) sense remotely the infra-red spectra emitted from the ocean to a depth of 20 microns with an accuracy of ±0.4°C (Weiss, 1966, p. 45). Thus this method is good only for sea surface temperature measurements. However as the sensors do not have to be physically in contact with the ocean and can be focused to scan narrow beams, radiation thermometers are used now in aircraft and artificial satellites for making sea surface temperature maps for large areas as shown for example in Fig. II for the North Atlantic.

Thermistors are used (1) singly as in the XBT or in the attached temperature-depth salinity probe (TDS -- discussed in salinity section) which measures continuous profiles as the sensor moves through the water column or (2) in groups in a thermistor chain. The thermistor chain at present consists of 34 thermistors attached 25 feet apart on a 830 foot chain (Grafa, 1967). The chain is towed through the water and the weight of the chain keeps the thermistors arrayed in a concave down catenary. At a speed of six knots the base of the chain tows at about 750 feet below the surface. The thermistors are interrogated in sequence from the surface down so that each thermistor is read when it is approximately under the initial position of the surface thermistor giving essentially a vertical profile of temperature for each interrogation cycle.

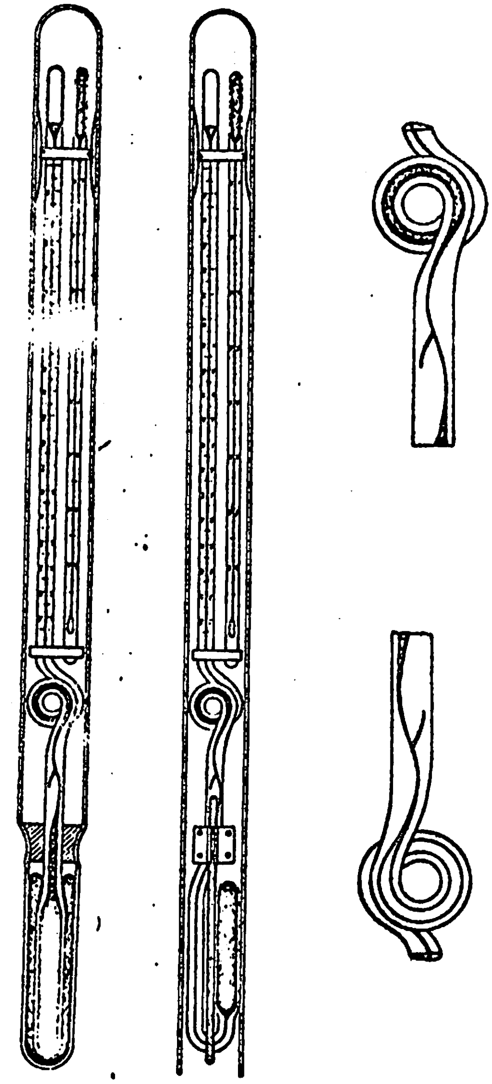
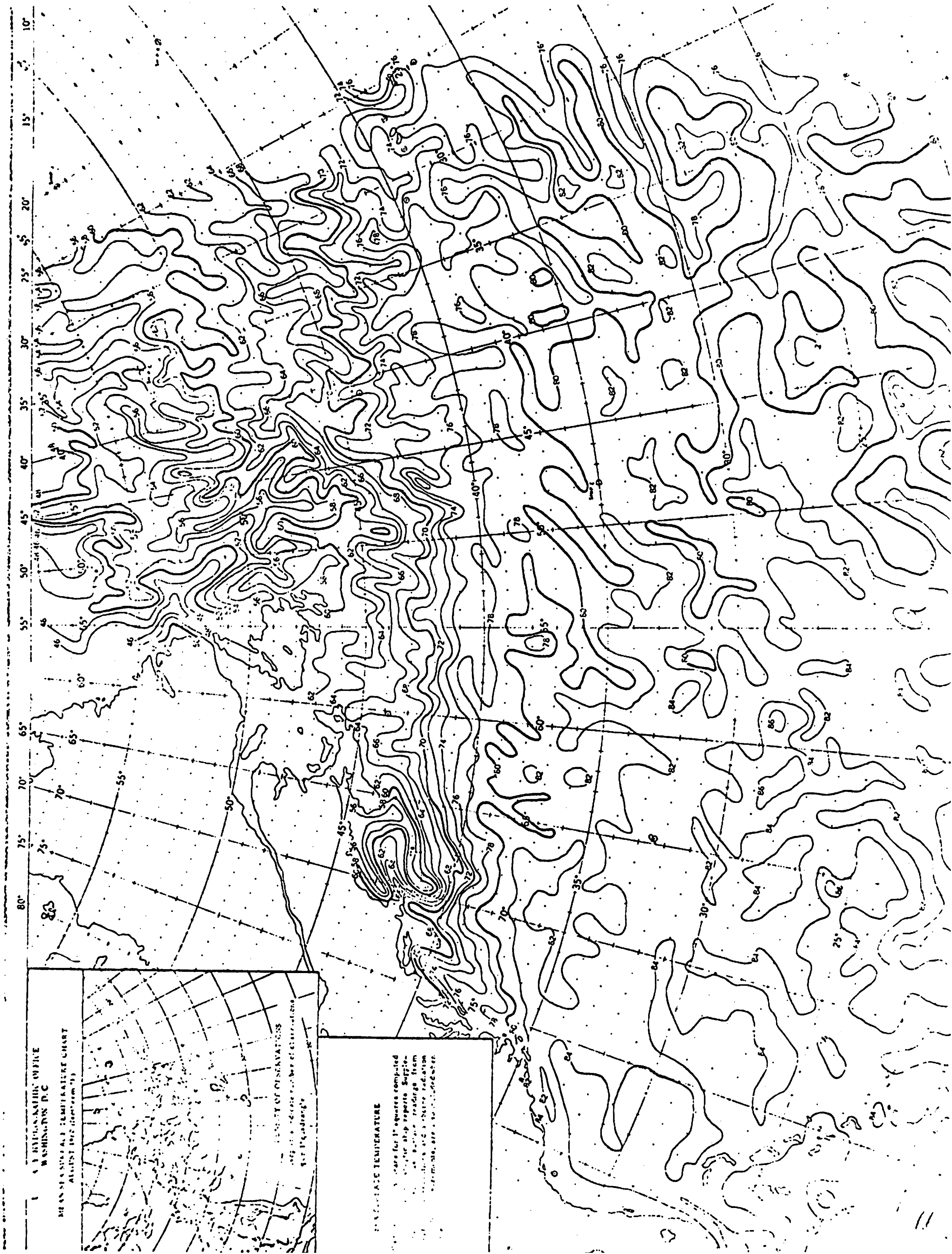


Fig. 86. Protected and unprotected reversing thermometers in set position, that is, before reversal. To the right is shown the constricted part of the capillary in set and reversed positions.



GUIDE TO REMOTE SEA TEMPERATURE MEASURING INSTRUMENTS

Company	Model	Type/ Installation	Availability	Detector	Features	Price
Barnes Engineering Company	PRT-5	Radiometer/ airborne	Off the Shelf	Germanium immersed thermistor bolometer	High sensitivity (o.o5C). Self-contained batteries or a-c operated. Ruggedized modular packaging. Narrow and wide fields of view (0.14, 2, 20 degrees).	$6,950.00
	PRT-4a	Radiometer/ airborne	Off the Shelf	Germanium immerced thermistor bolometer	High sensitivity (0.15C) Rugged modular packaging. Choice of fields of view (2 or 20 degrees)	$4,135.00
	IT-3	Radiometer/ airborne	Off the Shelf	Thermistor bolometer	Good sensitivity (0.5C) Commercial Packaging. Field of view 3 degrees.	$1,975.00
	14-322 (AN/AAR-31)	Radiometer/ airborne	Custom	Thermistor bolometer	High sensitivity (better than 0.1C) Meets MIL-E-5400. Maximum ruggedness, stability, reliability, designed for pod mounting. Field of view 2.2 degrees.	Based upon special requirements
	14-430	Radiometer/ airborne	Custom	Germanium immersed thermistor bolometer	High sensitivity (0.5C) Weight 2.9 lb. Field of view 0.55 degrees, Ruggedized for modern booster launch.	Based upon special requirement
	14-432	Radiometer, dual channel/ satellite	Custom	Germanium immersed thermistor bolometer	High sensitivity (0.5C) Weight 4.6 lb, Field of view 2 independant fields, 0.33 degree, Ruggedized for modern booster launch.	Based upon special requiremets
	Barnes/ Bofors	Real time infrared camera/ airborne	Off the Shelf	Indium antimonide, liquid nitrogen cooled	TV-like image on cathode ray tube. Wide-angle field of view (25x12.5 degrees). 125 line image at four frames per second. High sensitivity (0.1C)	$21,975.0

CT 201 A

TEMPERATURE REFERENCES

Grafa, J. E., 1967, The Thermistor Chain, Undersea Technology, V. 8, No. 5, pp. 28-32.

Robertson, E. C., Raspet, R., Swartz, J. H., and Ullard, M. E., 1966, Properties of Thermistors Used in Geothermal Investigations, U. S. Geol. Survey Bull., 1203-B.

Spilhaus, A. F., 1938, A Bathythermograph, Jour. Marine Res., Vol. 1, pp. 95-100.

Weiss, M., 1968, Remote Temperature Sensing, Oceanology Internat., Vol. 3, No. 6, pp. 44-48.

Saunders, P. M., 1967, Aerial Measurement of Sea Surface Temperature in the Infrared, Jour. Geophys. Res., Vol. 72, p. 4109-4117.

Steinhart, J. S. and Hart, S. P., 1968, Calibration curves for Thermistors, Deep Sea Res., Vol. 15, pp. 497-503.

MAUL, G.A., AND SIDRAN, M., 1973, ATMOSPHERIC EFFECTS ON OCEAN SURFACE TEMPERATURE SENSING FROM NOAA SATELLITE SCANNING RADIOMETER: JOUR. GEOPHYS. RES. OCEANS AND ATMOSPHERES, V. 78, p. 1909-1916.

UNIVERSITY OF CALIFORNIA
CE 201A Fall 1971

Department of Civil Engineering
Div. of Hydraulic and Sanitary Engr.
Instructor: Pat Wilde

SALINITY

UNITS: Parts per thousand, grams per kilogram, parts per mille

DEFINITIONS: For titration measurement SAL 0/00 = 1.805 CL 0/00 + 0.03
For conductometric measurement SAL 0/00 = 1.8066 CL 0/00

Where CL = chlorinity = number in grams of atomic weight silver just necessary to precipitate the halogens: chlorine, bromine, ~~chlorine~~, in 0.3285233 kilograms of sea water. Florine, iodine

Of the parameters used in physical oceanographic studies probably only salinity is unique to the profession. Position, temperature, and pressure, obviously are utilized by other scientific disciplines. The closest analog to salinity used in other fields is concentration. As an approximation the salinity may be considered as the total dissolved solids in a marine sample. However, precisely salinity is a defined parameter which developed as a function of the methods used to measure it. *

Salinity, although ideally a function of density, is measured practically (1) chemically by argentometric titration of the halogens of sea water (Knudsen, 1901 and Harvey, 1963, p. 125-130); (2) electrically by the conductivity of sea water (Cox and others, 1967), and (3) less commonly, optically by index of refraction.

Historically, the concept of salinity gradually developed from the attempts to measure the constituent chemicals in sea water. An early pioneer in the field was Robert Boyle whose work "Observations and Experiments on the Saltness of the Sea" in 1670 commenced modern studies of sea water. Boyle first used silver nitrate to test for sea water, noting that river and lake waters did not give a white precipitate with $AgNO_3$ whereas sea water did. Boyle tried to determine the weight of solids in sea water by evaporation of a known volume of sea water but could not reproduce his results. He eventually used a hydrometer to measure specific gravity which proved a much more accurate way to determine the salt content. An excellent summary of the historical development of physical and chemical oceanography is found in Riley (1965).

The modern concept of salinity stems from the work of an international commission to study the interrelationship among chlorinity, salinity, and density of sea water which culminated in Knudsen's (1901) hydrographic tables. The standard definitions of chlorinity and salinity were given by Forch and others (1902). These definitions and methods were chemically derived from argentometric titrations of sea water with silver nitrate. In general, they are based on the concept developed by Forchhamer and demonstrated by Dittmar (1884) that the major dissolved constituents of sea water were in constant ratio to each other regardless of concentration. This means that one only need to measure one constituent as the amounts of the remaining constituents may be derived by calculations.

For sea water the easiest ions to measure are the halogens so the definition of salinity is based on silver titration of the halogens. The most abundant halogen in sea water is chlorine so total halogen content of sea water was defined as the chlorinity.

Salinity (in parts per thousand) = 1.8050 chlorinity + 0.03

The original definition of chlorinity as the total halogens was modified because it depended on the atomic weights of silver, chlorine, and bromine which are revised periodically. The new definition of chlorinity is in terms of the weight of silver necessary to precipitate the halogens.

As Lyman and Fleming (1940) noted, the definition of salinity is derived from the halogens only so that there is a deviation of salinity from actual total dissolved solids. As further revised by Lyman (1959):

Total dissolved solids (TDS) = 1.8110 chlorinity + 0.069

so that the value of salinity is somewhat less than the total dissolved solids.

The Knudsen titration method used standard water whose chlorinity had been precisely determined by comparison with primary standards kept at Copenhagen. Such water is available in 300 ml ampules from

I.A.P.O.
Standard Sea Water Service
Charlottenlund Slot, Denmark.

These standards called "Copenhagen Water" or "normal water" are distributed at low cost through support of the Carlsberg Foundation.

Although the titration technique has been used extensively in physical oceanography it has obvious disadvantages for shipboard use. The main problem is the difficulty of reading the meniscus of the buret on a rolling ship and the individual error in determining the end-point. Electrochemical end-point determinations have eliminated some uncertainty in such titrations, but have added to the bulk of equipment needed. In general, precise chemical work is to be avoided at sea and should only be attempted when storage errors are greater than operator errors.

An alternate method using the conductivity of sea water for the measurement of salinity has developed since the 1920's with the development of electronics, and is rapidly replacing titration as the standard shipboard method. The conductivity method has the advantage that few manipulations are needed to obtain a value and no great skill is necessary. An excellent discussion of the conductivity method is given by Cox (1965). Two types of conductivity measurements have been used: (1) with electrodes and (2) by induction. The electrode method has calibration drift problems because of possible polarization and fouling of the electrodes. Such problems are eliminated by the induction method in which the sea water solution is made a conducting loop in a transformer. Brown and Hamon (1961) have built the most popular shipboard type of induction salinometer described in Figs. 1, 2 and 3. The induction method is further attractive for shipboard use as only about 50 ml of sample is needed and the sample does not come in contact with the coils which are embedded in a salt resistant plastic. With the development of reliable induction conductivity methods salinity was redefined for conductivity measurements (Cox, 1966) in terms of conductivity ratios (sea water compared to standard sea water samples) as:

Salinity (parts per thousand) = 1.80655 chlorinity.

Actually as

$$\begin{aligned} \mathrm{SAL}\ 0/00 = & -0.08996 + 28.29720R_{15} \\ & + 12.80832R_{15}^{2} - 10.67869R_{15}^{3} \\ & + 5.98624R_{15}^{4} - 1.32311R_{15}^{5} \end{aligned}$$

Figure 1

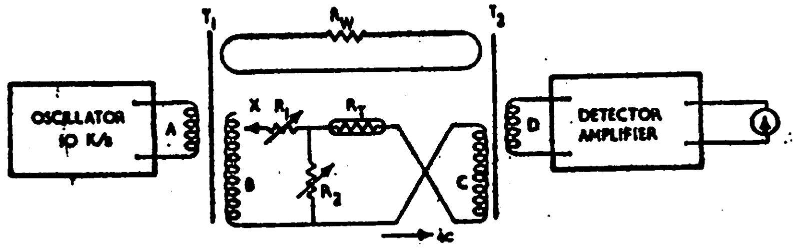


FIG. 1. Simplified circuit diagram of the inductive salinometer. T_1: voltage transformer. T_2: current transformer. R_w: resistance of the water path linking T_1 and T_2. R_1: standardizing resistor. R_2: adjustment for temperature compensation. R_T: thermistor. X: main balancing control (conductivity ratio, details in FIG. 2).

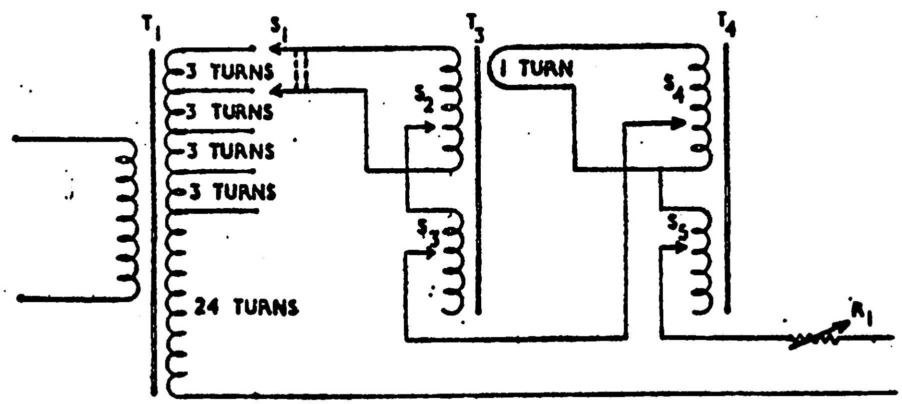


FIG. 2. Details of the tapped transformer ratio adjustment. S_1: 4 position switch (ratios 0·8, 0·9, 1·0, 1·1). S_2–S_5: decade ratio switches. The windings on T_3 and T_4 have the following numbers of turns: S_2 and S_4: 10 × 10 turns; S_3 and S_5: 10 × 1 turn.

From Brown and Hamon (1961, p. 66)

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Figure 2

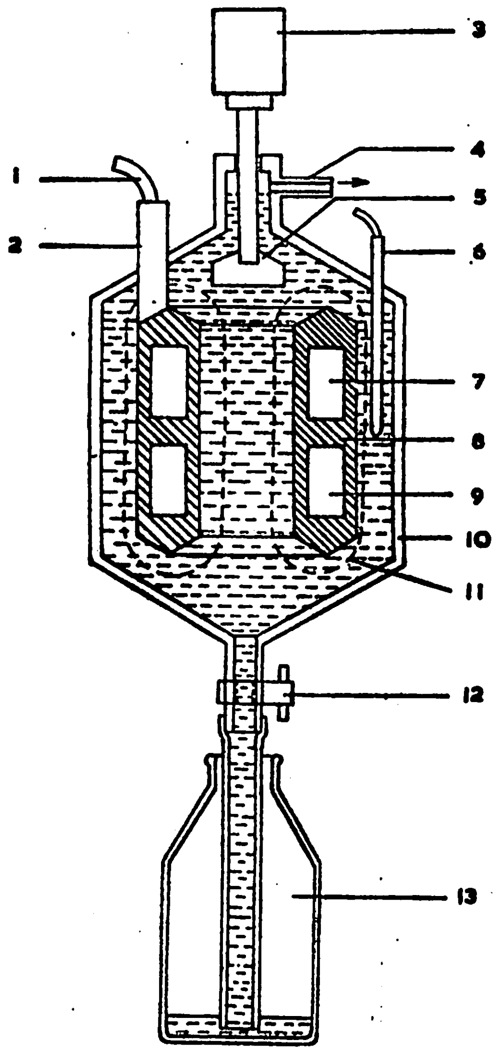


FIG. 3. Simplified diagram of the measuring head. 1 : leads from toroid assembly. 2 : support stem for toroid assembly. 3 : stirring motor. 4 : connection to aspirator. 5 : stirrer. 6 : thermistor. 7 : toroidal core of voltage transformer. 8 : toroid assembly. 9 : toroidal core of current transformer. 10 : clear plastic housing. 11 : path of electric current in the water sample. 12 : stopcock. 13 : sample container.

From Brown and Hamon (1961, p.67)

Figure 3

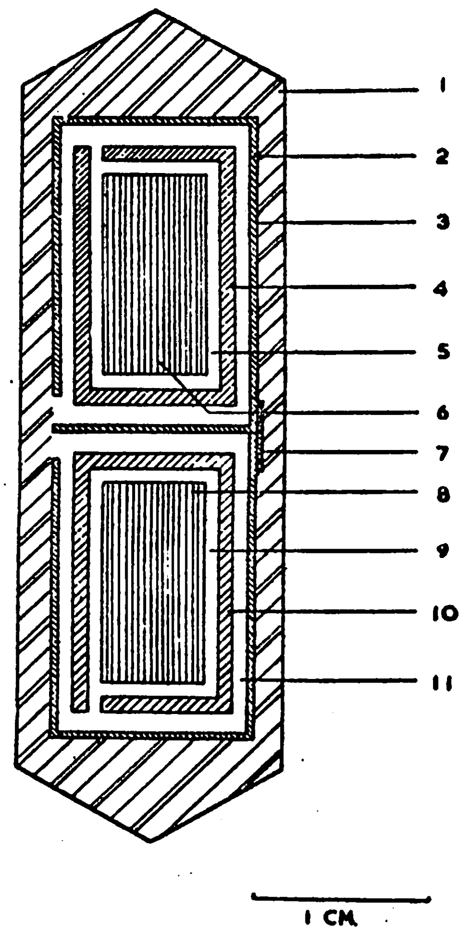


FIG. 4. Section of one side of toroid assembly. 1 : epoxy resin casitng. 2 : electrostatic shield (brass). 3 : secondary winding of voltage transformer (B, FIG. 1). 4 : magnetic shield (copper). 5 : primary winding of voltage transformer (A, FIG. 1). 6 : strip-wound core of voltage transformer (Permalloy 'C'). 7 : section of heater winding. 8 : strip-wound core of current transformer. 9 : detector winding (D, FIG. 1). 10 : magnetic shield. 11 : primary winding of current transformer (C, FIG. 1).

From Brown and Hamon (1961, p.68)

Where R_{15} = conductivity ratio at 15° C

$$= \frac{\text{conductivity of sample}}{\text{conductivity of 35 0/00 SAL.}}$$

Induction salinometers are calibrated with Copenhagen Water so tables of salinity-conductivity ratios were developed by UNESCO (see Cox, 1966). The above salinity-chlorinity relationship was developed by measurements on samples from various oceans as shown in Fig. 4.

The errors in conductivity measurements of salinity are due to temperature and pressure variations. As shown in Figs. 5 and 6 the major effect is caused by temperature. Weyl (1964) developed an empirical relationship between specific conductance (millimhos per centimeter):

Temperature (0-25° C) and chlorinity (17-20 0/00):

$$\log K_s = 0.57627 + 0.892 \log Cl\ (0/000)$$
$$-10^{-4}\tau[88.3 + 0.55\tau + 0.0107\tau^2 - Cl\ (0/00)\ (0.145 - 0.002\tau + 0.0002\tau^2[$$

where K_s = specific conductance

$\tau = 25 - \tau^{\circ}$ centigrade.

Studies of the pressure effects by Horn and Frysinger (1963) (Fig. 5) show essentially linear relationships with pressure for low salinity waters and for open ocean waters to about 1000 bars (10,000 meters) where the curve flattens. Fig. 6 shows that the observed values were much greater than the calculated values suggesting an increase in conductance. As Fig. 6 shows no such change for NaCl solutions, the deviation is due to some other compound present in sea water. Horne and Frysinger (1963) note that ionization of weak electrolytes increase with pressure and suggest Mg^{++} and SO_4^{++} dissociation.

In any case, such a discussion of the chemistry of sea water is beyond the scope of physical oceanography. However, the value of these deviations are important for calibration in situ salinity probes (see Fig. 7) which are becoming more attractive and because in situ measurements to great depths are more common.

Figure 4

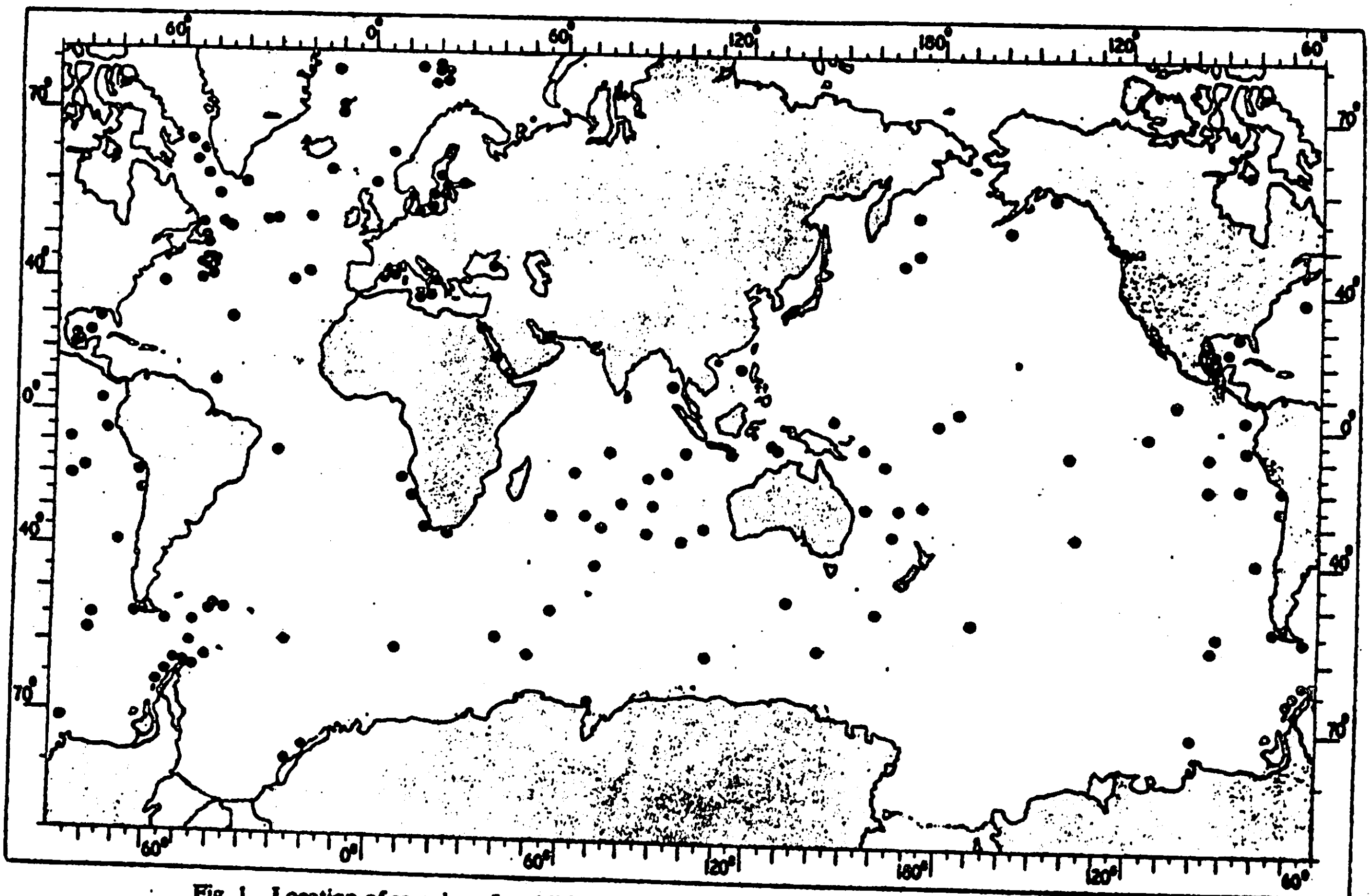


Fig. 1. Location of samples. In addition there were samples from 82° N 151° W and 82° N 105° E.

From Cox and Others (1967, p. 205)

Figure 5

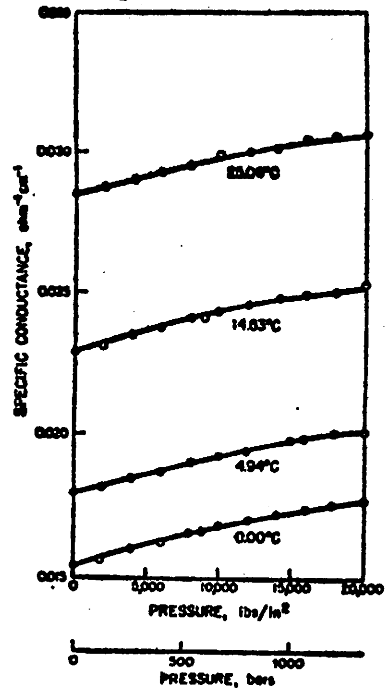


Fig. 3. Specific conductance of 9.680 per mil chlorinity sea water.

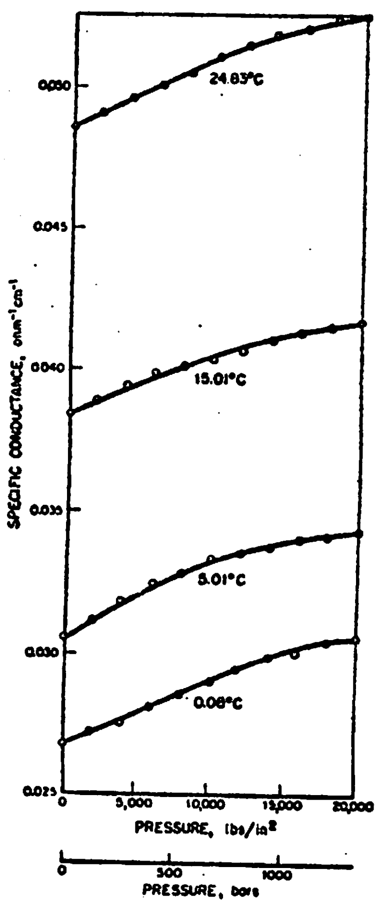


Fig. 2. Specific conductance of 17.61 per mil chlorinity sea water.

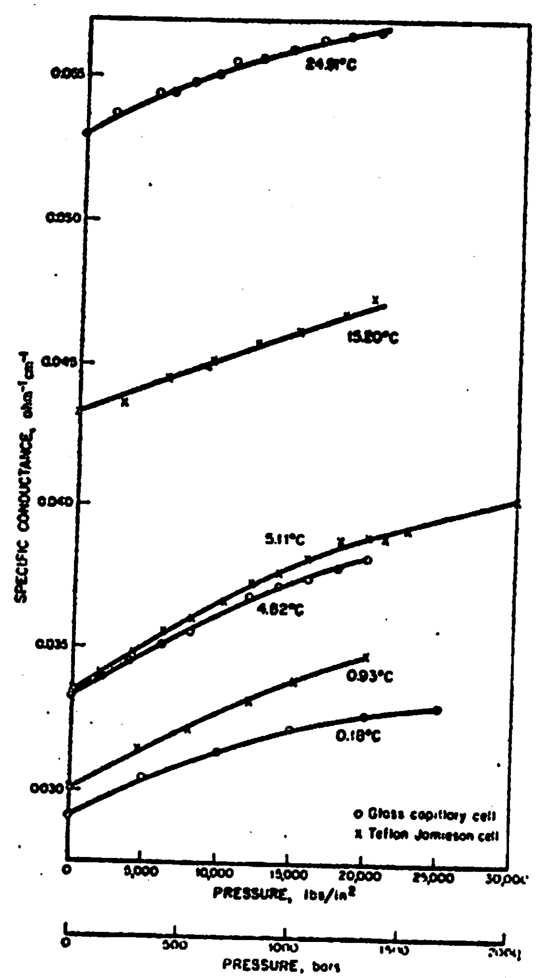


Fig. 1. Specific conductance of 19.376 per mil chlorinity sea water.

From Horne and Frysvger (1963, 1970, 1971)

Figure 6

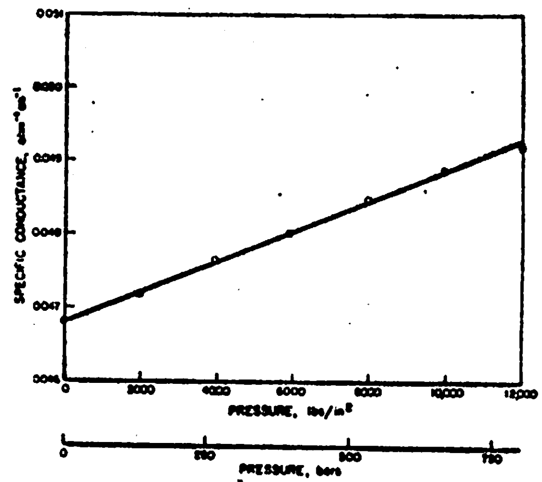


Fig. 7. Specific conductance of 0.50 *M* NaCl.

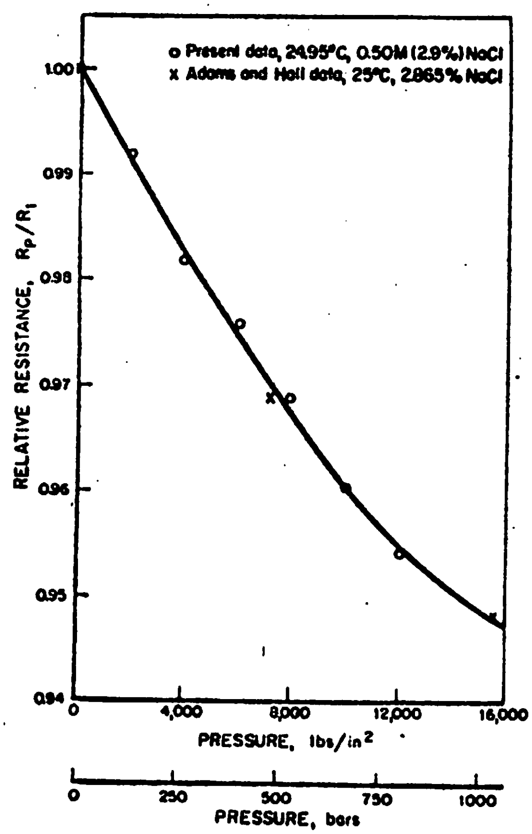


Fig. 6. Relative resistance of sodium chloride solutions.

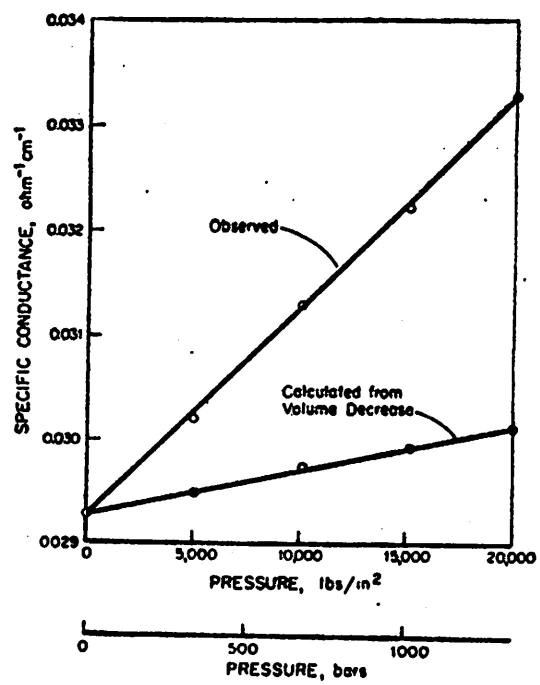


Fig. 5. Comparison of observed and calculated specific conductances of 35 per mil salinity sea water at 0°C.

From Horne and Fryswger (1963, p. 1971, 1972)

Figure 6

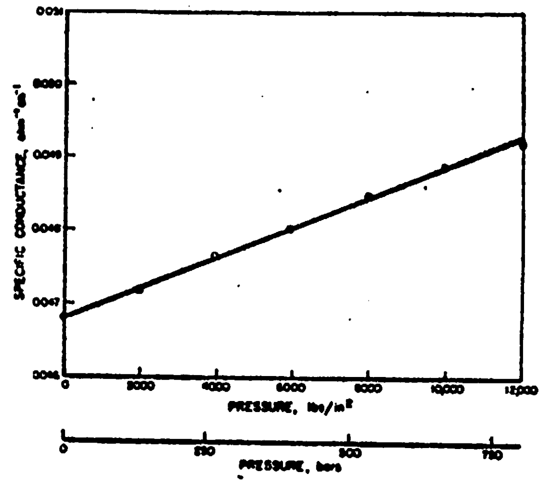


Fig. 7. Specific conductance of 0.50 *M* NaCl.

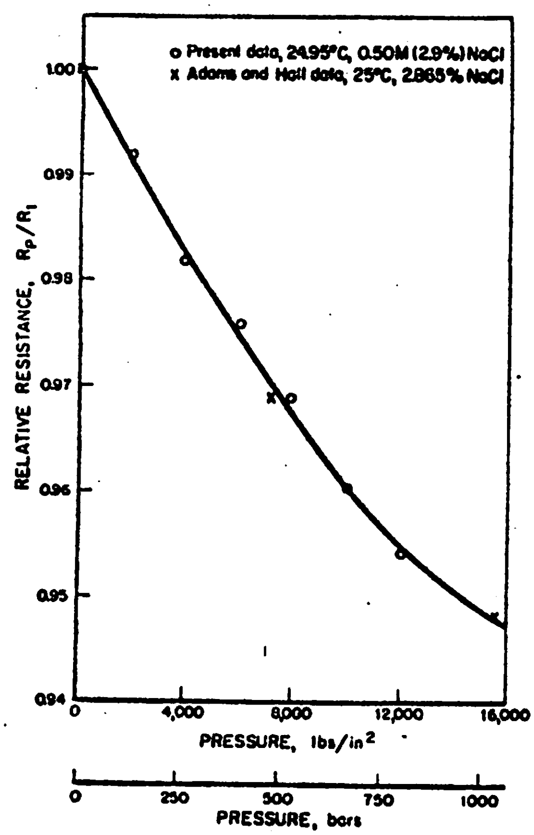


Fig. 6. Relative resistance of sodium chloride solutions.

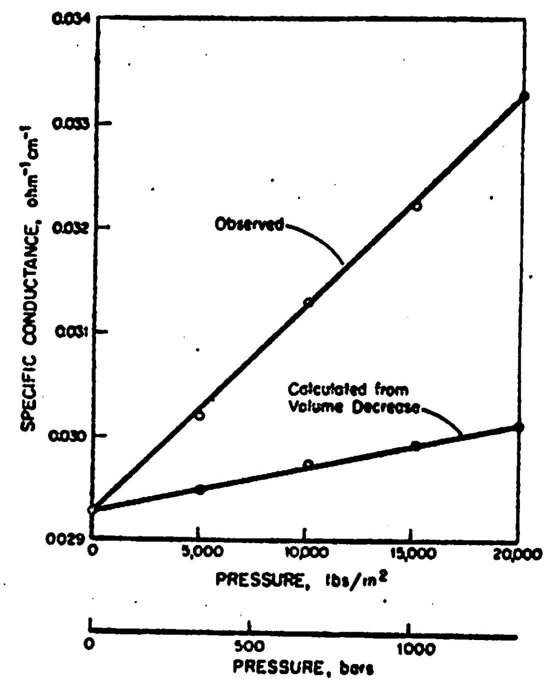


Fig. 5. Comparison of observed and calculated specific conductances of 35 per mil salinity sea water at 0°C.

From Horne and Fryswger (1963, p. 1971, 1972)

Figure 7

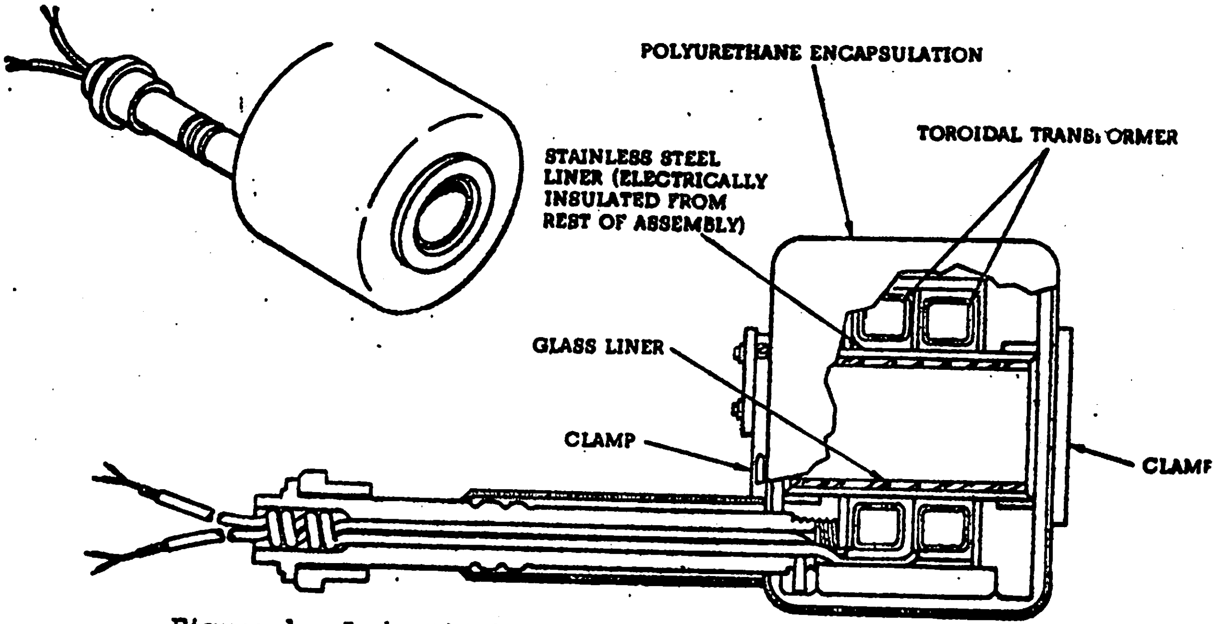
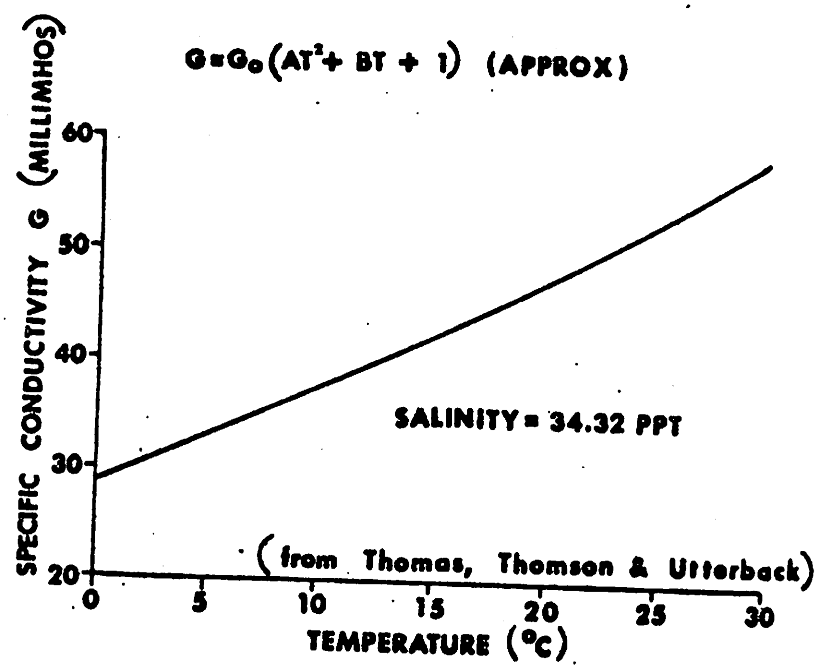
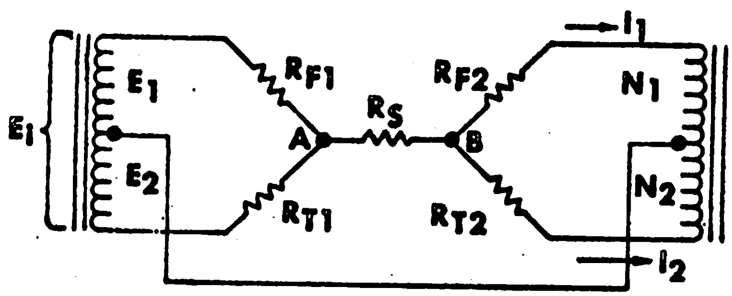


Figure 1. Inductively-Coupled Conductivity Sensor

From Brown (1968, p. 564)

Figure 2. Conductivity as a Function of Temperature





$R_{F1} = R_{F2}$

$R_{T1} = R_{T2}$

$E_1 = E_2 = \frac{1}{2} E_I$

$N_1 = N_2$

$I_o = I_1 - I_2$

R_{T1} & R_{T2} ARE THE PLATINUM RESISTANCE THERMOMETERS

Figure 3. Double Bridge Temperature Compensation Circuit

REFERENCES

Brown, N. L., 1968, An In Situ Salinometer for Use in the Deep Ocean: Marine Science Instrumentation, V. 4, p. 563-577.

Brown, N. L., and Hamon, B. V., 1961, An Inductive Salinometer: Deep-Sea Research, V. 8, p. 65-75.

Cox, R. A., 1965, The Physical Properties of Sea Water, in Chemical Oceanography, V. 1, Riley and Skirrow (Eds.): London and New York, Academic Press, p. 73-120.

Cox, R. A., 1966, International Oceanographic Tables: UNESCO, Table Ia, For Converting Conductivity Ratio at 15°C to Salinity of Sea Water, p. 8.

Cox, R. A., McCartney, M. H. and Culkin, F., 1970, The Specific Gravity/Salinity/Temperature Relationship in Natural Sea Water: Deep-Sea Research, V. 17, p. 679-689.

Dittmar, W., 1884, Report on Researches into the Composition of Ocean Water Collected by the H.M.S. Challenger during the years 1873-1876: Voy. Challenger Physics and Chemistry, V. 1, p. 1-251.

Harvey, H. W., 1963, The Chemistry and Fertility of Sea Waters: Cambridge, Eng., Cambridge Univ. Press, 240 p.

Knudsen, M., 1901, Hydrographic Tables: G. E. C. GAD, Copenhagen, 63 p.

Lyman, J., and Fleming, R. H., 1940, Composition of Sea Water: Jour. Mar. Research, V. 3, p. 134-145.

Lyman, J., 1959, Chemical Considerations; in Conference on Physical and Chemical Properties of Sea Water: Nat. Acad. Sci. Nat. Res. Council Pub. 600, p. 87-97.

Riley, J. P., 1965, Historical Introduction, in Chemical Oceanography, V. 1,Riley and Skirrow (Eds.): London and New York, Academic Press, p. 1-41.

Sverdrup, H. V., Johnson, M. W., and Fleming, R. H., 1942, The Oceans: Englewood Cliffs, N. J., Prentice-Hall, 1087 p.

Thompson, T. G., 1932, Physical Properties of Sea Water. Physics of the Earth, V. 5, Oceanography: U.S. Nat. Res. Council, Bull. 85, p. 63-94.

Weyl, P. K., 1964, On the Change in Electrical Conductance of Sea Water with Temperature: Limnol. Oceanogr., V. 9, p. 75-78.

UNIVERSITY OF CALIFORIA
CE 201 A

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Acoustic Properties

Vibrations in the band 1H_Z to 100 KH_Z or approximately the human audible range (5 H_Z to 20 KH_Z) are used in oceanography for (1) depth ranging as in the sonic depth recorder, (2) location of objects in the water column as in sonar, and (3) navigation by transponders.

Longitudinal waves such as sound are elastic waves with a propagation velocity given by the LaPlace Equation

[S.H. - quantity of heat necessary to raise the temp of 1 gr. of a substance 1°C]

$$V = \sqrt{\frac{Cp/Cv}{\rho\kappa}}$$

where: Cp = specific heat of medium at constant pressure
Cv = Specific heat of medium at constant volume
ρ = density of medium
κ = a diabatic compressibility

where:

$$\kappa = \frac{\mu + P d\mu/dP}{1-\mu P}$$

P = Pressure
μ = Coefficient of isothermal compressibility

in sea water Cp/Cv and ρ are about one so their ratio is approximately unity thus: $V \sim \sqrt{\frac{1}{\kappa}}$ or primarily a function of temperature and pressure.

The thermal profile of the ocean, in general, is characterized by three layers: (1) a surface mixed isothermal layer, (2) a thermocline, and (3) a deep isothermal layer. However, as the ocean is density stratified, pressure increases almost linearily with depth. EVERY 10 M → 1 ATMOS. This combination of stepwise decrease of temperature with depth and linear increase of pressure produces a sound velocity profile which (1) decreases with depth with decreasing temperature to the base of the main thermocline But (2) increases with depth below the base of the main thermocline in the deep isothermal layer where the pressure effects become dominant.

The presence of a sound velocity minimum at the base of a thermocline produces interesting refraction effects. By the law of refraction

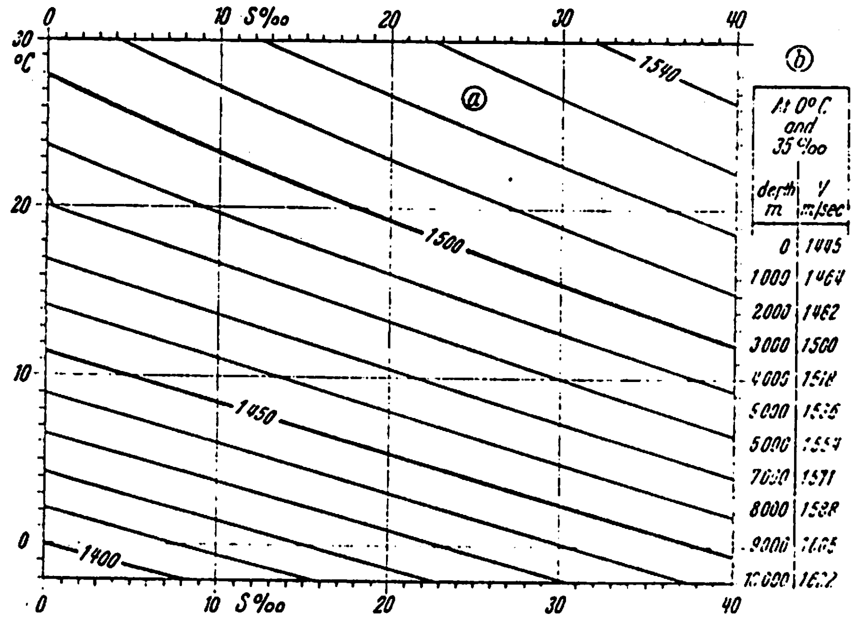
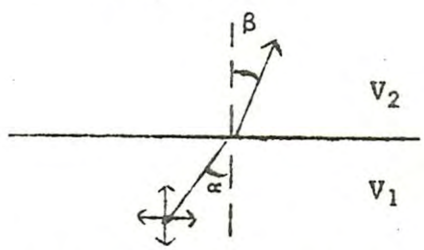


Fig. 27. a, Velocity of sound V (in m. per sec.) in sea water as a function of temperature and salinity, neglecting pressure; b, velocity of sound V at different depths at a temperature of 0°C. and a salinity of 35‰ as a function of pressure. (a and b after S. Kuwahara, 1938.)

(Snell's Law): $\frac{\text{SIN } \alpha}{\text{SIN } \beta} = \frac{V_1}{V_2}$ where: α = angle of incidence
β = angle of refraction
V_1 = sound velocity in layer of propagation
V_2 = sound velocity

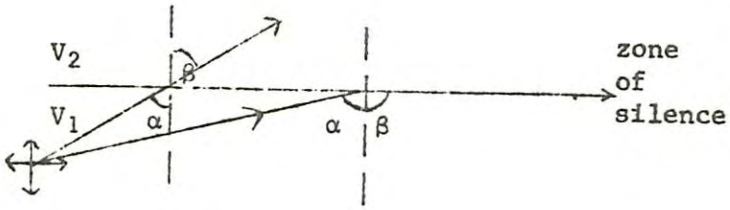
SIN α > SINβ or their ratio is always greater than one so total reflection ($\beta = 90^o$, SINβ = 1) can not occur thus all sound waves can leave the layer of propagation except those parallel to the layer boundary.

For $V_1 > V_2$:



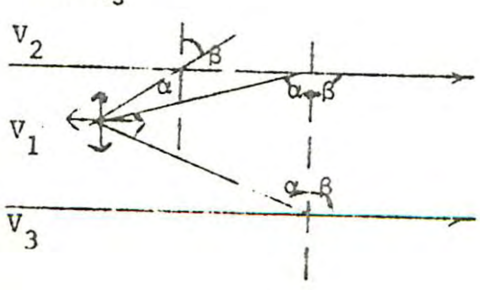
SIN α < SIN β or their ratio is always less than one so for certain angles β can be 90^o or greater and total reflection can occur causing a zone of silence.

For $V_1 < V_2$:



This situation would be the case for sound propagated just below the base of the main thermocline or transient thermoclines in the mixed layer. The correlation of the zone of silence explains why temperature profiles obtained almost instantaneously by Bt's are used extensively in anti-submarine warfare, as submarines could take advantage of the zone of silence and remain undectected by sonar reflections sent from a surface ship.

$V_1 < V_2$
$V_1 < V_3$



For a three layer case which is the idealized situation at the sound velocity minimum, much of the sound energy is contained in the initial zone of propagation as total reflection occurs both above and below the zone. Thus the zone of minimum sound velocity at the base of the main thermo-

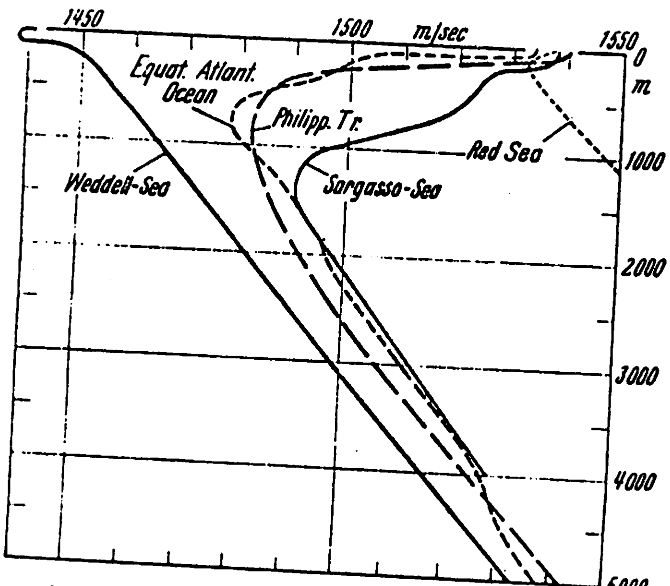


Fig. 28. Examples of horizontal velocity of sound in the world ocean. (According to G. Dietrich, 1952.)

CE 201 A

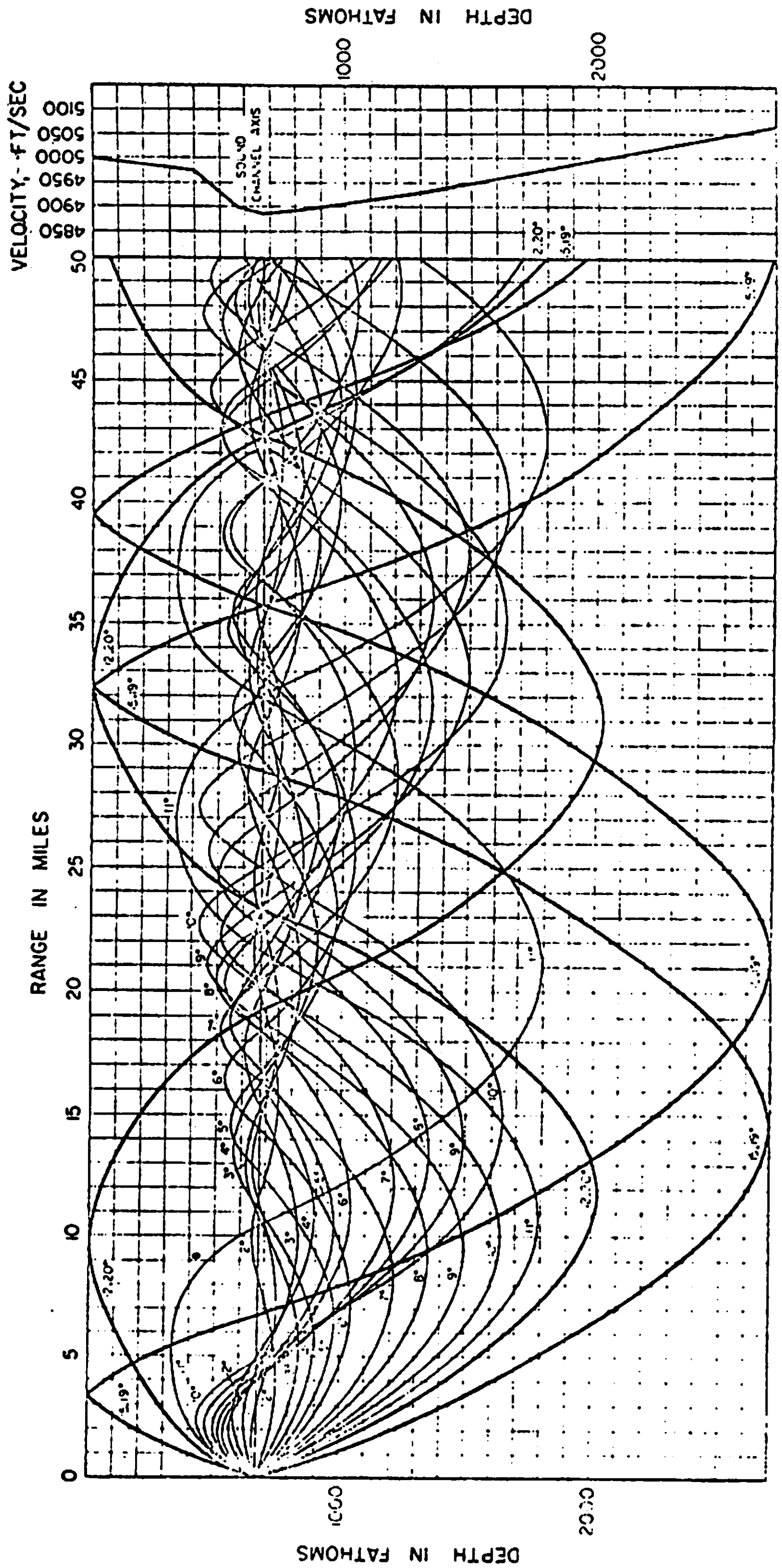


Fig. 16.5 Ray paths in deep-sea sound channel (after Ewing *et al.*, 1946).

cline acts as a wave guide often called the sound channel.

The thermal pressure profile is essentially geometrically similar throughout the world ocean so the sound channel is continuous over large areas. Therefore sound waves propagated in the sound channel may travel long distances. This phenomenon forms the basis of the sofar (sound fixing and ranging)system used for determination of the location of distress calls at sea. A distressed vessel or a downed pilot drops a charge pre-set usually a pressure trigger, to explode in the sound channel. The sound waves are received, by various hydrophones placed in the sound channel throughout the world ocean, in sequence as a function of the distance from the explosion to the receiving hydrophone.

The usefulness of sound in sea water is determined by how far sound can travel and be detected which is a function of its absorption in sea water or:

$$Jx = J_o e^{-2vx}$$

$$v = \frac{8}{3}\,\frac{\pi^2 \mu f^2}{\rho V^3}$$

where:
J_o = source intensity
Jx = intensity at some distant x
v = coefficient of sound absorption
f = frequency

With the velocity of sound in sea water approximately fixed at 1500 meters/sec, sound absorption increases with f^2 so low frequency sound travels farther than high frequency. Ideally very low frequency would be the most useful for sound ranging at sea. However the practical limitation is the wave length at the particular frequency which determines the size of the sensor needed to detect the sound waves.

$\lambda = v/f$ λ = wave length

Frequency Hz	Wave Length	Comment
1000	1.5M	Impractical
10000	15 cm	Used in Transponder Navigation
12000	12.5cm	U.S. frequency
30000	5cm	used by Germans

Unfortunately low frequency sound waves require large diameter sensors which are impractical for most oceanographic ships.

Albers, V. M., (ed). 1962, Underwater Acoustics, U.I.: New York, Plenum Press.

_______ ____________________, 1967, Underwater Acoustic, V.II: New York Plenum Press, 416 p.

Dietrich, G., 1963, General Oceanography: New York, Interscience, p. 67-73,

McLellan, H. J., 1965, Elements of Physical Oceanography: Oxford, Eng. Pergamon Press, Chapter 16, Sound Propagation, p. 117-124.

Tolstoy, I. and Clay, C. S., 1966, Ocean Acoustics: New York, McGraw - Hill, 293 p.

Tucker, D. G. and Gazey, B. K., 1966, Applied Underwater Acoustics: Oxford, Eng, Pergamon, 244 p.

are causd by suspended particles and dissolved organics. Open ocean water after filtration of suspended matter shows extinction curves similar to that of pure water. However filtered coastal water shows a shift in the minimum extinction wave length towards yellow. This selective absorption may be caused by dissolved organics (humus) and ferrous salts from rivers and decaying plankton which has been called gelbstoffe or yellow substances (Kalle, 1938). Turbidity measurements (difference between the measured co-efficient of extinction and that of pure water measured in the red wave length) show an inverse relation to salinity near shore which indicates that the shift is related at least partially to fresh water influx.

The color of sea water, neglecting cloud cover effects, or for clear skies with the optical center of incident light at 0.47 μ is (1) blue for clear open ocean water where extinction is due to the absorptive properties of water alone·(2) blue green to green with the addition of organic particles; near coral reefs: (3) yellow with high organic content off rivers with high humic content -- Yellow Sea. When particles are large enough to produce individual light reflecion then the sea color is determined partially by the color of the particles or (4) chocolate brown for muddy waters near shore and (5) green, brown or red for high concentrations of plankton as in red tide plankton blooms.

REFERENCES:

Dietrich, G., 1963, General Oceanography: New York, Interscience, p. 73-83.

Jerlov, N. G., 1951, Optical Studies of Ocean Water: Repts. Swedish Deep-Sea Exped., Phys. Chem. V. 3 (1) p. 1-59.

__________, 1963, Optical Oceanography: Oceanogr. Mar. Bio. Ann. Rev., V1., p. 89-114.

Kalle, K., 1938, Zum Problem Der Meerwasserfarbe: Ann. Hydrogr. v. Marit. Meterol., V. 66 (5), p. 1-13.

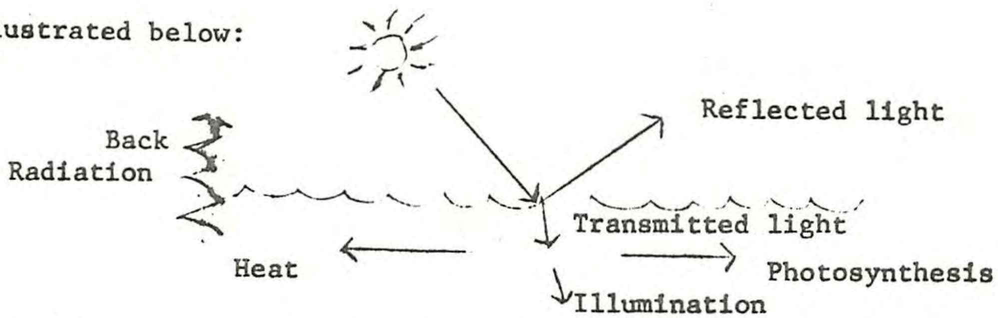
McLellan, H. J., 1965, Elements of Physical Oceanography: Oxford, Eng. Pergamon, Chap. 17, Light in the Sea. p. 125-130.

UNIVERSITY OF CALIFORNIA
CE 201 A

Department of Civil Engineers
Div., Hydraulic and Sanitary Eng.
Instructor: Pat Wilde

OPTICAL PROPERTIES

The interaction between incident light waves and sea water is complex as illustrated below:



Of major concern to the physical oceanographer are the illuminative properties of light which determine (1) the depth to which photosynthetic plants can live, (2) refractive index, (3) extinction of light with depth, (4) transparency and/or turbidity, and (5) the color of sea water.

Radiant flux is lost in sea water through attenuation of light due to (1) absorption as (a) sea water is denser than air and (b) by particulates and (2) scattering by various sized particles in sea water.

Ideal extinction of light in pure water follows Lambert's Law:

$$J = Jo \times 10^{-ad}$$

J = intensity at depth d
Jo - initial intensity
a = coefficient of extenction
d = 1 meter (by convention in oceanography)

Plots of a versus wave lenght in pure water show an absolute minimum in blue light or 0.46 to 0.48 μ. The infrared waves particularily are asborbed. This explains why at depth objects illuminated by natural light at the surface have a blue cast. In pure sea water the extinction coefficients change little from those of pure water even with variations in temperature and salinity.

The major variations of the extinction coefficient from those in pure water

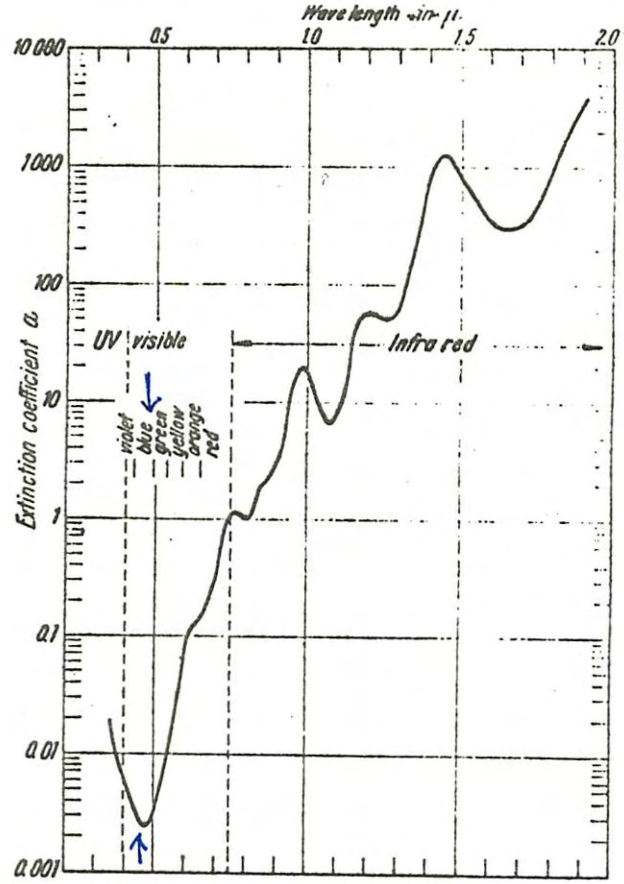


Fig. 32. Extinction coefficient a for a light beam in pure water as a function of its wavelength (thickness of layer, 1 m.). (According to G. Dietrich, 1939.)

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CE 201A, Fall 1971

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THERMODYNAMIC APPROACH TO PHYSICAL OCEANOLOGY

Part I

Basic equations in terms of primary measured parameters temperature, pressure, and concentration (salinity).

The goal of Physical Oceanography is the understanding and ellucidation of the physical properties of sea water. Thus it would be advisable initially to describe theoretical thermodynamic concepts in terms of parameters which may be measured practically in sea water. In this manner (1) the attainable limits of accuracy are defined, (2) areas of further research are identified, and (3) the discipline is developed in a systematic logical manner.

Any thermodynamic function J usually can be described in terms of temperature, pressure and composition as

$$dJ = \left(\frac{\partial J}{\partial T}\right)_{P,m_i,m_j} dT + \left(\frac{\partial J}{\partial P}\right)_{T,\, m_i,\, m_j} dP$$

$$+ \sum_i \left(\frac{\partial J}{\partial m_i}\right)_{T,P,m_j} dm_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

In oceanography as the oceans have significant depth an additional term must be added to allow for variations in gravity or $Md\Phi$ ....(2)

Where:

m_i = i component of the mass

M = total mass

Φ = gravity or geopotential

For discussions of problems at constant or small depth changes this term may be ignored (see Fofnoff, 1962, p.5)

Terms such as $\left(\frac{\partial J}{\partial m}\right)_{T,P,m_i}$ are called partial mass quantities (if m is given as moles as is the case in chemical thermodynamics such terms are called partial molar quantities. Mass terms are used in physical oceanography as they are easier to use in dynamic calculations). An extremely useful partial mass quantity is the chemical potential defined by Gibbs as:

$$\mu_i = \left(\frac{\partial E}{\partial m_i}\right)_{N,V,m_j} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Where

E = internal energy

N = entropy

Accordingly a generalized first law of thermodynamics would be

$$dE = TdN - PdV + \Sigma\, \mu_i dm_i + Md\Phi \text{ ....} (4)$$

Converting to specific quantities, that is per unit mass, by dividing by M equation (4) becomes, ignoring depth changes:

$$d\overline{E} = Td\overline{N} - Pd\overline{V} + \sum_i \mu_i d\overline{X}_i \text{ ....} (5)$$

when $\overline{X}_i = mi/M$

and the Gibbs Equation

$$G = E + PV - TN \text{ ....} (6)$$

Becomes: Diff 5

$$d\overline{G} = d\overline{E} + PdV + VdP - Td\overline{N} - \overline{N}dT \text{ ....} (7)$$

Or

$$d\overline{G} = - \overline{N}dT + \overline{V}dP + \Sigma\, \mu_i d\overline{x}_i \text{ ....} (8)$$

Thus any thermodynamic calculations for any multi-component system such as sea water must consider not only temperature (T), Pressure (P) and Concentration (X) But also the chemical potential (μ). Calculation of X and μ for each component in sea water for every sample would be an impossible task. However, in sea water as demonstrated by Dittmar (1884) and re-affirmed by Culkin and Cox (1966) and Morris and Riley (1966), the major components in sea water are in essentially constant proportion to each other regardless of the addition or subtraction of H_2O. Thus for each

Sea Water is conservative

$$\overline{X}_i = \frac{m_i}{M} \quad \text{.......................................} (9)$$

there is a constant λ_i so that

$$\overline{X}_i = \lambda_i \, \overline{S} \quad \text{.......................................} (10)$$

Where

$\overline{S}$= A number related to the total dissolved solids or the total concentration of solids in sea water

In oceanography such a number is called the salinity (sal) and is given in parts per thousand or grams per kilogram rather than grams per grams (S).

salinity

$$\overline{S} = 10^{-3} \text{ SAL} = 1.8065 \times 10^{-3} \text{ CL} \quad \text{.............} (11)$$

Where:

CL = the chlorinity of the sample.

The constant of proportionality (λ) is called the ion/chlorinity ratio

$$\lambda_i = (m_i/10^3)\ /\text{CL} \quad \text{...........................} (12)$$

In essence, use of the salinity-chlorinity concepts in thermodynamic calculations, treats all the dissolved species as a single solute, so a combined chemical potential may be used

$$\mu_s = \Sigma\, \lambda_i \mu_i \quad \text{...............................} (13)$$

And

$$\mu_w = \text{Chemical potential of } H_2O \quad \text{..............} (14)$$

So

$$\mu = \mu_s - \mu_w \quad \text{.......................................(15)}$$

then equation (8) becomes:

$$d\overline{G} = -\overline{N}dT + \overline{V}dP + \mu d\overline{S} \quad \text{..........................(16)}$$

The independent variables of equation (16) temperature, pressure, and salinity may be measured directly with little error. However the coefficients specific entrophy, specific volume, and specific chemical potential can not be measured directly and must be calculated by the use of appropriate formulae.

Taking the first derivatives of equation (16)

$$\frac{\partial\overline{G}}{\partial T} = -N \quad \text{..(17)}$$

$$\frac{\partial\overline{G}}{\partial P} = \overline{V} \quad \text{...(18)}$$

$$\frac{\partial\overline{G}}{\partial S} = \mu \quad \text{...(19)}$$

ENTROPHY

As shown in equation (1) entrophy may be represented by:

$$d\bar{N} = \frac{\partial\bar{N}}{\partial T}dt + \frac{\partial\bar{N}}{\partial P}dP + \frac{\partial\bar{N}}{\partial S}dS \quad \ldots\ldots\ldots\ldots\ldots (20)$$

As C_P specific heat at constant pressure =

$$C_P = \left(\frac{dQ}{\partial T}\right)_P \quad \ldots\ldots\ldots\ldots\ldots (21)$$

and

$$d\bar{N} = \frac{dQ}{dT} \quad \ldots\ldots\ldots\ldots\ldots (22)$$

then the first term

$$\frac{\partial\bar{N}}{\partial T} = \frac{C_P}{T} \quad \ldots\ldots\ldots\ldots\ldots (23)$$

or

$$\bar{N} = \int\frac{CP}{T}dT \quad \ldots\ldots\ldots\ldots\ldots (24)$$

As $$\frac{\partial\bar{G}}{\partial T} = -\bar{N} \quad \ldots\ldots\ldots\ldots\ldots (17)$$

then $$\frac{\partial\bar{N}}{\partial P} = \frac{-\partial^2\bar{G}}{\partial T\,\partial P} \quad \ldots\ldots\ldots\ldots\ldots (25)$$

Since $$\frac{\partial\bar{G}}{\partial P} = \bar{V} \quad \ldots\ldots\ldots\ldots\ldots (18)$$

then the second term of equation (20) is

$$\frac{\partial\bar{N}}{\partial P} = \frac{-\partial\bar{V}}{\partial T} \quad \ldots\ldots\ldots\ldots\ldots (26)$$

In like manner using equation (19) the third term

$$\frac{\partial\bar{N}}{\partial S} = \frac{-\partial^2\bar{G}}{\partial T\,\partial S} = \frac{-\partial\mu}{\partial T} \quad \ldots\ldots\ldots\ldots\ldots (27)$$

so equation (20) becomes

$$d\bar{N} = \frac{C_P}{T}dT - \frac{\partial\bar{V}}{\partial T}dP - \frac{\partial\mu}{\partial T}dS \quad \ldots\ldots\ldots\ldots\ldots (28)$$

All terms except μ can be measured so that $\bar{N}$ may be determined by integration of (28) with μ approximated.

SPECIFIC VOLUME - very important

In oceanography the specific volume $\frac{V}{M} = \bar{V}$ is symbolized by (α) which also is the inverse of the specific gravity (ρ) or
(density)

$$\alpha = \frac{1}{\rho} \text{ in cubic centimeters/gram} \quad \ldots\ldots\ldots\ldots (29)$$

So by equation (1) the specific gravity relationships may be written:

$$d\alpha = \frac{\partial\alpha}{\partial T}dT + \frac{\partial\alpha}{\partial P}dP + \frac{\partial\alpha}{\partial S}dS \quad \ldots\ldots\ldots\ldots (30)$$

where $\left(\frac{\partial\alpha}{\partial T}\right)_{P,S}$ = thermal expansion . . . . (31)

$\left(\frac{\partial\alpha}{\partial P}\right)_{T,S}$ = isothermal compressibility . . . . (32)

$\left(\frac{\partial\alpha}{\partial S}\right)_{P,T}$ = saline contraction . . . . (33)

Equation (30) forms the basis for the sea water equation of state as specific volume may be defined in terms of state variables.

A complete formulation of the specific volume as a function of temperature, pressure and salinity was done by Wilson and Bradley (1968). Their experimental apparatus and best fit formula is given in fig. (2) where υ = specific volume. (Page 74)

Of course, practically one is more accustomed to thinking in terms of density or specific gravity rather than specific volume. The inverse of specific volume (see table 1) or specific gravity is somewhat greater than one. Thus variations in the density field of interest to physical oceanographers are in the second decimal place or higher. So to avoid writing 1.0xxxx for each specific gravity value, a quantity sigma (σ) is used for specific gravity so that

$$\sigma_{sal,\tau,P} = (\rho_{sal,\tau,P} - 1)1000 \quad \ldots\ldots\ldots\ldots (34)$$ Specific Gravity

Such a quantity would be the in place or in situ value. Practically, for shipboard measurements, a simplification of the in situ σ is used called sigma-T (σ_T) (this should be written $\sigma_{sal,\tau}$ to be correct), which ignores the pressure effects in equation (30). As table 1 shows, the value of $\frac{\partial\alpha}{\partial P}$ is at least an order of magnitude less than the value of the salinity and temperature dependent co-efficients. Cox and others (1970) calculated σ_T using a precise but simple archemedian weighing device as shown in figs. (3, 4).

Table one gives the values of the specific volume and its derivatives for confitions of the standard ocean ($0^{\circ}C$, $35^{\circ}/oo$, 1 ATM).

Table One

SPECIFIC VOLUME AND ITS DERIVATIVES WITH RESPECT TO TEMPERATURE, SALINITY AND PRESSURE AT 0°C, 35‰ AND ATMOSPHERIC PRESSURE
(Pressure in Decibars)

α	$= 0.972643$	$\left(\frac{\partial^2\alpha}{\partial T^2}\right)_{p,s}$	$= +13.2 \times 10^{-6}$
$\left(\frac{\partial\alpha}{\partial T}\right)_{p,s}$	$= +50.1 \times 10^{-6}$	$\left(\frac{\partial^2\alpha}{\partial s^2}\right)_{T,p}$	$= +0.76 \times 10^{-6}$
$\frac{1}{\alpha}\left(\frac{\partial\alpha}{\partial T}\right)_{p,s}$	$= +51.5 \times 10^{-6}$	$\left(\frac{\partial^2\alpha}{\partial p^2}\right)_{T,s}$	$= +15.3 \times 10^{-12}$
$\left(\frac{\partial\alpha}{\partial s}\right)_{T,p}$	$= -762.5 \times 10^{-6}$	$\left(\frac{\partial^2\alpha}{\partial T\partial s}\right)_{p}$	$= +2.8 \times 10^{-6}$
$\left(\frac{\partial\alpha}{\partial p}\right)_{T,s}$	$= -45.3 \times 10^{-7}$	$\left(\frac{\partial^2\alpha}{\partial T\partial p}\right)_{s}$	$= +26.4 \times 10^{-11}$
$-\frac{1}{\alpha}\left(\frac{\partial\alpha}{\partial p}\right)_{T,s}$	$= +46.6 \times 10^{-7}$	$\left(\frac{\partial^2\alpha}{\partial s\partial p}\right)_{T}$	$= +14.8 \times 10^{-11}$

From FOFONOFF (1958)

Bradshaw and Schleicher (1970) have experimentally measured thermal expansion. Their apparatus and assumptions are shown in Figure 1.

Thermal expansion apparatus

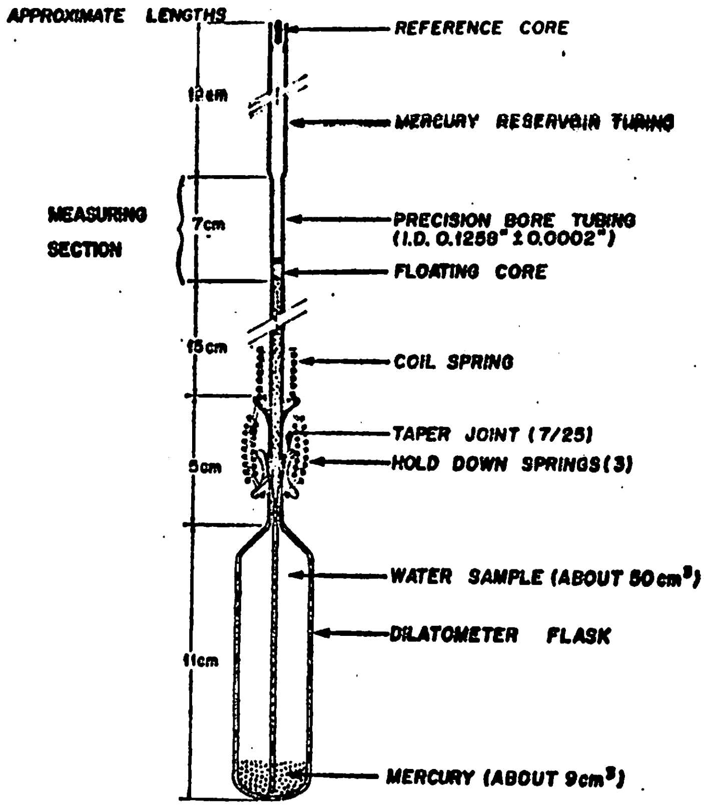

Fig. 1. The dilatometer is constructed of fused quartz. The larger diameter mercury reservoir tubing holds the excess mercury required by the compression of the water sample for the runs made above atmospheric pressure.

Equation

The equation used to calculate the change in volume of the sea water sample from the temperature and distance measurements was

$$V_{T,P,S}^{sw} - V_{0,P,S}^{sw} = (\bar{A}_{0,P} + K)(L_{0,P} - L_{T,P}) + \epsilon(L_{0,P}, L_{T,P}) + \bar{\alpha}_{T,P}^{0,P}$$
$$\times T\left[V_{0,P,L(0,P)}^{dil} + \frac{\bar{A}_{0,P}}{3}(3L_{0,P} - 2L_{T,P})\right] - M(v_{T,P}^{m} - v_{0,P}^{m}) \qquad (1)$$

where

P	= pressure, bars (absolute)
T	= temperature, °C
S	= salinity, ‰
$L_{T,P}$	= distance between floating and reference cores at T and P, cm
$V_{0,P,L(0,P)}^{dil}$	= volume of the contents of the dilatometer at $T = 0$, P and $L_{0,P}$, cm³
$\bar{A}_{0,P}$	= average area of the bore of measuring section of precision-bore tubing at $T = 0$ and P, cm²
$\bar{\alpha}_{T,P}^{0,P}$	= mean coefficient of thermal expansion (volume) of dilatometer from $T = 0$ to T at P, $(°C)^{-1}$
$V_{T,P,S}^{sw}$	= volume of sea water sample at T, P and S, cm³
$v_{T,P}^{m}$	= specific volume of mercury at T and P, cm³/g
M	= mass of mercury, g
K	= $\frac{\Delta V_{flask}}{\Delta L_{T,P}}$, where ΔV_{flask} is the dilation of the flask caused by an increase $(-\Delta L_{T,P})$ in mercury height
$\epsilon(L_{0,P}, L_{T,P})$	= a term, small compared with the volume change, which corrects for the non-uniformity of the area of the precision-bore tubing

In the derivation of the equation $\bar{\alpha}^2$ and $\bar{\alpha} \times K$ terms have been neglected. It has also been assumed that temperature effects on the geometry of the mercury surface in the precision-bore tubing and on the dimensions of the cores are insignificant*. The coefficients for length and area thermal expansion were taken to be 1/3 and 2/3, respectively, of that for volume thermal expansion. (The same assumption was made for compressibility as well as thermal expansion in subsequent calculations).

From Bradshaw and Schleicher
(1970, p. 692, 696-697)

Figure 2

Specific volume determinations apparatus

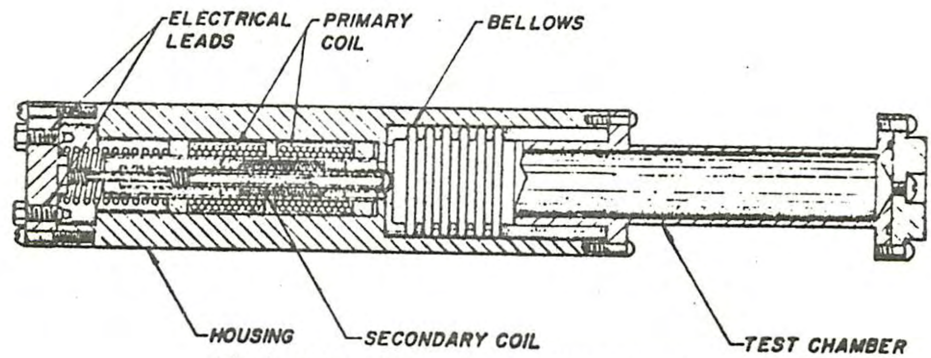

Fig. 1. Differential transformer with test chamber.

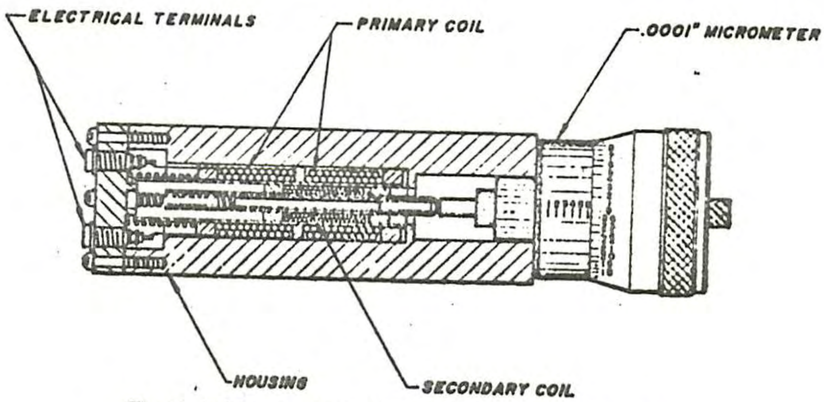

Fig. 2. Differential transformer with micrometer head.

$$v = 0{\cdot}70200 + \frac{100\,(17{\cdot}5273 + 0{\cdot}1101\,T - 0{\cdot}000639\,T^2 - 0{\cdot}039986\,S - 0{\cdot}000107\,TS)}{(P + 5880{\cdot}9 + 37{\cdot}592\,T - 0{\cdot}34395\,T^2 + 2{\cdot}2524\,S)} \quad (3)$$

In this equation P is expressed in bars, T is given in degrees centigrade and S is given in parts per thousand. The standard deviation of the equation from the 897 data points was 0·00013 cm^3/g.

From Wilson and Bradley (1968, p. 356,361)

74

Figure 3

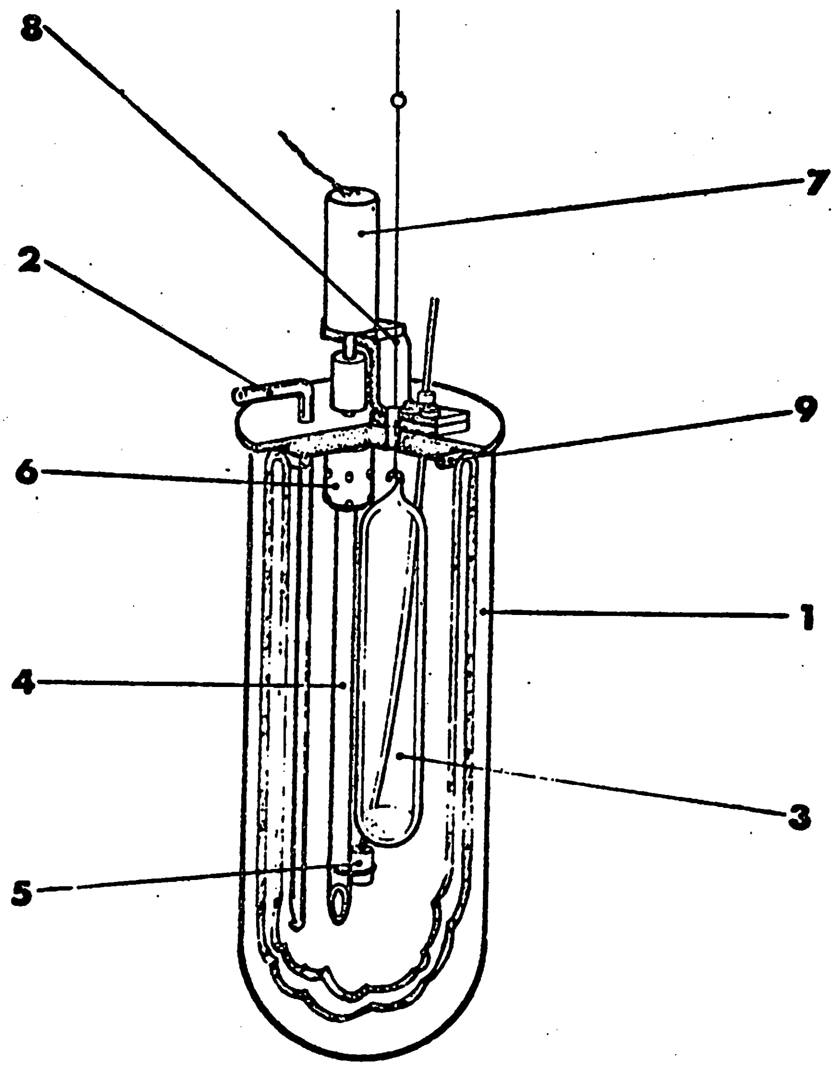

Fig. 1. Apparatus for determination of specific gravity of sea water.
1—Unsilvered vacuum flask
2—Capillary glass filling tube
3—Silica sinker
4—Silver tube
5—Quartz thermometer probe
6—Stirrer paddle housing
7—Stirrer motor
8—Pt–Ir wire
9—Neoprene "O" ring

From Cox and others (1970, p.681)

Figure 4

Determination of volume of sinker

The sinker was weighed in air, before and after attachment of the suspension. The weight of the sinker *in vacuo* was then calculated as follows:

$$\text{Weight of sinker } \textit{in vacuo} \ (W) = W_a\left(1 + \frac{\delta}{\rho} - \frac{\delta}{\rho_1}\right).$$

where W_a = weight of sinker in air

ρ = density of sinker (1·070 g/ml)

ρ_1 = density of weights (8·00 g/ml)

δ = density of air

= density of dry air at 760 mm

$$\times \left(\frac{\text{barometric pressure (mm Hg)} - \text{relative humidity factor}}{760}\right)$$

Relative humidity factor = 0·375 × vapour pressure of water. Then volume of sinker at t°C

$$(V_t) = \frac{(\text{weight of sinker } \textit{in vacuo}) - (\text{weight of sinker in pure water at } t^\circ\text{C})}{\text{density of pure water at } t^\circ\text{C}}.$$

For calculation of V_t from this expression the table of densities of pure water given by TILTON and TAYLOR (1937) was used.

According to KNUDSEN (1901)

$$\sigma_t = (S_t - 1)\,1000,$$

where S_t is the specific gravity of sea water at t°C referred to pure water at 4°C.

$$\sigma_t = \left[\left(\frac{\text{Density of sea water at } t^\circ\text{C}}{\text{Density of pure water at } 4^\circ\text{C}}\right) - 1\right]1000$$

$$= \left\{\left[\left(\frac{\text{Weight of sinker } \textit{in vacuo} - \text{weight in sea water at } t^\circ\text{C}}{\text{Weight of sinker } \textit{in vacuo} - \text{weight in pure water at } 4^\circ\text{C}}\right) \times \left(\frac{\text{Volume of sinker at } 4^\circ\text{C}}{\text{Volume of sinker at } t^\circ\text{C}}\right)\right] - 1\right\} \times 1000.$$

From Cox and others (1970, p.684)

CHEMICAL POTENTIAL

Again by equation (1)

$$d\mu = \frac{\partial\mu}{\partial T}dT + \frac{\partial\mu}{\partial P}dP + \frac{\partial\mu}{\partial S}dS \quad \ldots\ldots\ldots\ldots\ldots\ldots (35)$$

By equations (17, 18, 19, 27 and 33)

$$\frac{\partial\mu}{\partial T} = \frac{\partial^2\bar{G}}{\partial T\,\partial S} = \frac{-\partial\bar{N}}{\partial S} \quad \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

$$\frac{\partial\mu}{\partial P} = \frac{\partial^2\bar{G}}{\partial P\,\partial S} = \frac{\partial\alpha}{\partial S} \text{ (saline contraction)} \quad \ldots\ldots\ldots (37)$$

By equation (15)

$$\frac{\partial\mu}{\partial S} = \frac{\partial\mu_s}{\partial S} - \frac{\partial\mu_w}{\partial S} \quad \ldots\ldots\ldots\ldots\ldots\ldots (38)$$

As

$$\mu_s = \sum_i \lambda_i \mu_i \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

$$\frac{\partial\mu_s}{\partial S} = \sum_i \frac{\lambda_i d\mu_i}{\partial S} + \sum_i \frac{\mu_i d\lambda_i}{\partial S} \quad \ldots\ldots\ldots\ldots (39)$$

But by equation (12)

$$d\lambda_i = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

For real solutions

$$d\mu_i = RT\, d\ell n\, \alpha_i \quad \ldots\ldots\ldots\ldots\ldots\ldots (41)$$

where α_i = activity of ion i = $\gamma_i \chi_i$

γ_i = activity coefficient

and

$$\mu_i = \mu_i^o + RT\ell n\alpha_i \quad \ldots\ldots\ldots\ldots\ldots\ldots (42)$$

where μ_i^o = chemical potential of pure component i.

As

$$\chi_i = \lambda_i S \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

$$\ell n\alpha_i = \ell n\, \gamma_i\lambda_i S \quad \ldots\ldots\ldots\ldots\ldots\ldots (43)$$

therefore

$$\frac{\partial\mu_i}{\partial S}_{P,T} = \frac{\partial\mu_i^o}{\partial S} + RT\ell n\gamma_i\lambda_i \frac{\partial\ell nS}{\partial S} + RT\ell n\gamma_i S \cdot \frac{\partial\ell n\lambda_i}{\partial S} + RT\ell n\lambda_i S \frac{\partial\ell n\gamma_i}{\partial S}$$

$$+ R\ell n\gamma_i\lambda_i S \frac{\partial T}{\partial S} \quad \ldots\ldots\ldots\ldots\ldots (44)$$

But

$$\partial \mu_i^o = 0 \quad \ldots\ldots (45)$$

$$\left(\frac{\partial T}{\partial S}\right)_{P,T} = 0 \quad \ldots\ldots (46)$$

and

$$\left(\frac{\partial \ell n \lambda i}{\partial S}\right) = 0 \quad \ldots\ldots (47)$$

For the range of sea water, equation (44) reduces to:

$$\left(\frac{\partial \mu i}{\partial S}\right)_{P,T} = \frac{RT\ell n\gamma i\lambda i}{S} + RT\ell n\lambda iS\left(\frac{\partial \ell n\gamma i}{\partial S}\right) \quad \ldots\ldots (48)$$

As Fig. (5) shows the activity coefficient certainly does change with concentration or as graphed ionic strength (I for sea water about 0.7). Whether γ changes in the region where λi is constant or in the range where the major ions are in constant proportion to each other is not known. If the various activity coefficients are constant then equation (48) further reduces to

$$\left(\frac{\partial \mu i}{\partial S}\right)_{P,T} = \frac{RT\ell n\gamma i\lambda i}{S} \quad \ldots\ldots (49)$$

So equation (39) becomes

$$\frac{\partial \mu S}{\partial S} = \frac{RT}{S}\sum_i \ell n\gamma i\lambda i \quad \ldots\ldots (50)$$

$\frac{\partial \mu w}{\partial S}$ may be measured by observing other secondary physical properties such as freezing point suppression (see Fofonoff (1962, p.17-22)) which is beyond the scope of this discussion - in summary the chemical potential variation with respect to temperature, pressure and salinity (eq. 35) may be represented as -

$$d\mu = \frac{-\partial \overline{N}}{\partial S}dT + \frac{\partial \alpha}{\partial S}dP + \left(\frac{RT}{S}\sum_i \ell n\gamma i\lambda i + \frac{\partial \mu w}{\partial S}\right)dS \quad \ldots\ldots (51)$$

$\boxed{\text{SUMMARY}}$

For Oceanographic purposes the useful free energy equation $dG = -\overline{N}dT + \alpha dP + \mu dS$ may be evaluated in terms of measurable quantities temperature, pressure, and salinity where :

$$d\overline{N} = \frac{CP}{T}dT - \frac{\alpha}{\partial T}dP - \frac{\partial \mu}{\partial T}dS \quad \ldots\ldots (28)$$

$$d\alpha = \frac{\partial \alpha}{\partial T}dT + \frac{\partial \alpha}{\partial P}dP + \frac{\partial \alpha}{\partial S}dS \quad \ldots\ldots (30)$$

$$d\mu = \frac{-\partial \overline{N}}{\partial S}dT + \frac{\partial \alpha}{\partial S}dP + \left(\frac{RT}{S}\sum_i \ell n\gamma i\lambda i = \frac{\partial \mu w}{\partial S}\right)dS \quad \ldots\ldots (51)$$

$-N\partial t$ = specific entropy
$\alpha \partial P$ = " volume
$\mu \partial S$ = " chem. potential

Figure 5

Showing the variation of individual activity coefficients with concentration

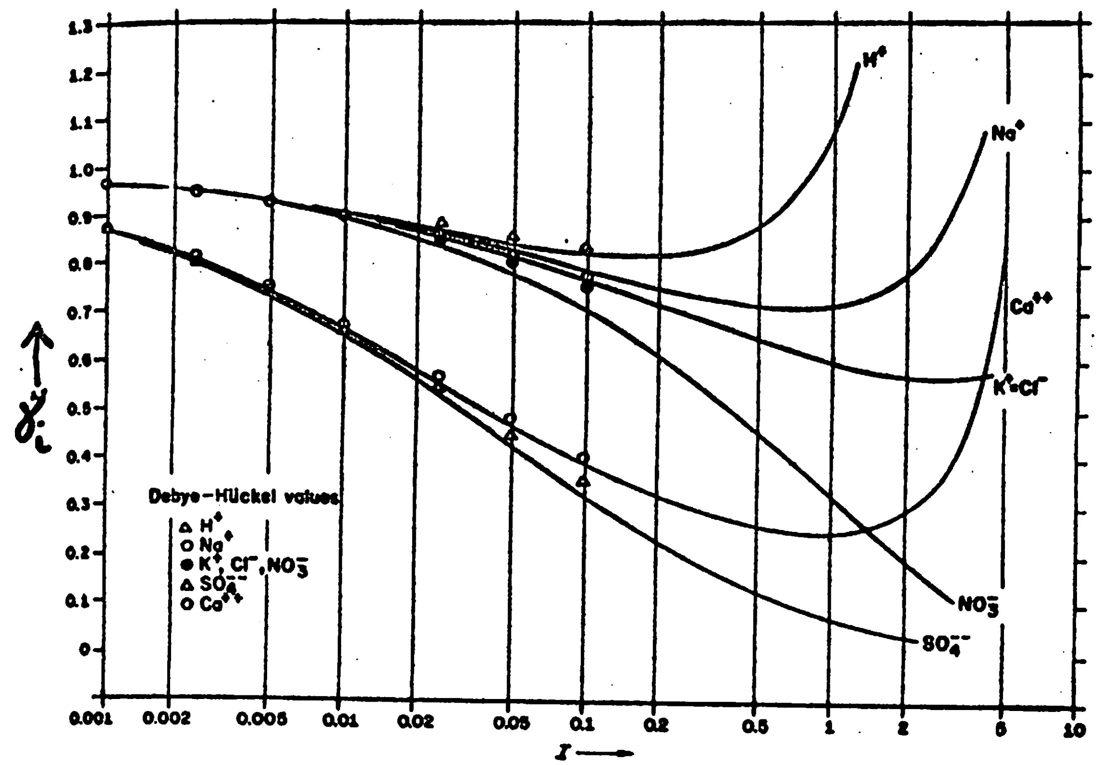

FIG. 2.15. Single ion activity coefficients vs. ionic strength for some common ions. Solid lines represent the values calculated by the mean salt method. Debye-Hückel values were calculated using equation (2.76), with $10^8/\mathring{a}_i = 9$ for H^+; 4 for Na^+; 3 for K^+, Cl^-, NO_3^-; 6 for Ca^{++}; and 4 for SO_4^{--}. The Debye-Hückel γ_i values for the monovalent ions converge, within experimental error, for $I < 0.01$.

Where

$$I = \tfrac{1}{2} \sum m_i \; z_i^2$$

$$Z = \text{charge}$$

From Garrels and Christ (1965, p.63)

All terms except $\frac{\partial \mu}{\partial T} = \frac{-\partial \bar{N}}{\partial S}$ can be evaluated exactly. Such a mathematical treatment assumes that the major ions in sea water are in constant proportion to each other so that sea water can be modeled as a fluid with a single solute represented by the salinity.

CE 201 A
Fall 1971

REFERENCES

Carritt, D. E. and J. H. Carpenter, 1958. The composition of sea water and the salinity-chlorinity-density problems. Physical and chemical properties of sea water. Nat. Acad. Sci.-Nat. Res. Council, Publ. 600, p.67-86.

Cox, R. A. and N. D. Smith, 1959. The specific heat of sea water. Proc. Roy. Soc. London, A252, p.51-62.

Cox, R. A., 1966, International Oceanographic Tables: UNESCO, Table Ia, For Converting Conductivity Ratio at 15°C to Salinity of Sea Water, p. 8.

Cox, R. A., McCartney, M. H. and Culkin, F., 1970, The Specific Gravity/Salinity/Temperature Relationship in Natural Sea Water: Deep-Sea Research, V. 17, p.679-689.

Craig, H., 1960. The thermodynamics of sea water. Proc. Nat. Acad. Sci., V. 46, p. 1221-1225

Culkin, F. C., and Cox, R. A., 1966, Sodium, Potassium, Magnesium, Calcium, and Strontium in Sea Water: Deep-Sea Res., V. 13, p. 789-804

Dittmar, W., 1884, Report on Researches into the Composition of Ocean Water Collected by the H.M.S. Challenger during the years - 1873-1876: Voy. Challenger Physics and Chemistry, V. 1, p. 1-251.

Eckart, C., 1958. Properties of water, Part III. The equation of state of water and sea water at low temperatures and pressures. Amer. J. Sci., V. 256, P. 225-240.

Ekman, V. W., 1908. Die Zusammendruckbarkeit des Meerwassers. Publ. Circ. Cons. Explor. Mer, No. 43, p. 1-47.

Fleming, R. H. and Revelle R., 1939, Physical Processes in the Ocean, In Recent Marine Sediments, Trask (ed.): Tulsa, Am. Assoc. Petrol. Geologist, p. 48-141.

Fofonoff, N. P., 1958. Interpretation of oceanographic measurements - thermodynamics. Physical and chemical properties of sea water. Nat. Acad. Sci.-Nat. Res. Council, Publ., 600, p. 38-66.

Fofonoff, N. P., 1962, Physical Properties of Sea Water, in the Sea, Hill (ed.): Interscience-Wiley, New York, V. 1, p. 3-30.

Forsch, C., M. Knudsen and S. P. L. Sorensen, 1902. Berichte uber die Konstantenbestimmungen zur Aufstellung der Hydrographischen Tabellen. Kgl. Danske Videnskab. Selskabs, Skifter, Naturvidenskab math., Afdel. XIII, p. 1-151.

CE 201 A
Fall 1971

Garrels, R. M. and Christ, C. L., 1965, Solutions, Minerals, and Equilibrium: New York, Harpers and Row, 450 p.

Klotz, I. M., 1958, Chemical Thermodynamics: Englewood Cliffs, N. J., Prentice-Hall, 369 p.

Knudsen, M., 1901, Hydrographic Tables: G. E. C. GAD, Copenhagen, 63 p.

Lyman, J. and Fleming, R. H., 1940, Composition of Sea Water: Jour. Mar. Research, V. 3, p. 134-145.

Lyman, J., 1959, Chemical Considerations; in Conference on Physical and Chemical Properties of Sea Water: Nat. Acad. Sci. Nat. Res. Council Pub. 600, p. 87-97.

Morris, A. W. and Riley, J. P., 1966, The Bromide/Chlotinity and Sulphate/Chlorinity Ratio in Sea Water: Deep-Sea Res., V. 13, p. 699-705.

Newton, M. S. and Kennedy, G. C., 1965, An Experimental Study of the P-V-T-S Relations of Sea Water: Jour. Marine Res., V. 23, p. 88-103.

Sverdrup, H. V., Johnson, M. W., and Fleming, R. H., 1942, The Oceans: Englewood Cliffs, N. J., Prentice-Hall, 1087 p.

Thompson, T. G., 1932, Physical Properties of Sea Water. Physics of the Earth, V. 5, Oceanogrpahy: U.S. Nat. Res. Council, Bull. 85, p. 63-94.

Wilson, W. and Bradley, D., 1968, Specific Volume of Sea Water as a Function of Temperature, Pressure and Salinity: Deep-Sea Research, V. 15, p. 355-363.

Thermodynamic Approach to Physical Oceanography Part II

Derived Paramenters

(A) Adiabatic Assumptions

Most physical oceanography data is obtained from hydrographic or Nansen casts by which a water sample and the water temperature is obtained from various depths. Until the recent development of in situ instrumentation all measurements except temperature were made at the surface usually on shipboard. By the nature of the Nansen bottle used to obtain the sample, raising the sample to the surface is essentially an adiabatic process or no heat or salt is exchanged with the environment. Thus for adiabatic conditions:

$$dQ = 0 = ds \quad \ldots \ldots \quad (52)$$

and

$$d\bar{N} = \frac{C_P}{T}\,dT - \frac{\partial\bar{v}}{\partial T}dP - \frac{\partial u}{\partial T}ds \quad \ldots \ldots \quad (20)$$

becomes, with respect to pressure

$$\frac{d\bar{N}}{dP} = \frac{C_P}{T}\left(\frac{\partial T}{\partial P}\right)_{N,S} - \left(\frac{\partial \alpha}{\partial T}\right)_{T,S} \quad \ldots \ldots \quad (52)$$

As $d\bar{N} = 0$ when $dQ = 0$, the effect of pressure on temperature by rearranging eq (52) is:

$$\left(\frac{\partial T}{\partial P}\right)_{N,S} = \frac{T}{C_P}\left(\frac{\partial \alpha}{\partial T}\right)_{P,S} \quad \ldots \ldots \quad (53)$$

This term is called the adiabatic temperature gradient and is designated

$$\left(\frac{\partial T}{\partial P}\right)_{N,S} = \Gamma \quad \ldots \ldots \quad (54)$$

Suitable integration of eq (54) solving for T can yield the temperature a packet of water reaches when the water is raised adiabatically to the surface. This temperature without the pressure effect is the potential temperature θ or

$$\theta = t_i + \int_{P_i}^{0} \Gamma\,(t^1,P,s)\,dP \quad \ldots \ldots \quad (55)$$

where

t_i = in situ temperature

P_i = in situ pressure

$$t^1 = t_i + \int_{P_i}^{P} \Gamma\ (t,P,s)dP$$

Potential temperatures are used in studies of water movement. For example, if the oceans were completely mixed ds/dz and dt/dz both = 0 all but the pressure effects on density vanish. As water is essentially incompressible the principle effect on density is from adiabatic temperature gradient. By comparison with the real ocean (A) if the real temperature gradient is less than Γ, or θ as calculated by EQ(55) increases with depth the water column is unstable. Such water if depressed adiabatically will sink as it is colder than the underlying water and if moved upward will rise to the surface as it would be warmer thus less dense (see Defant 1961, p.127). In the real case such water would most likely mix vertically (B) If the real temperature is equal to or greater than Γ the column is stable vertically. However, if potential temperatures are plotted rather than in situ temperatures horizontal water movements may be detected as shown in Fig. (6) because at any given depth flow should be from lower θ to higher θ (or from higher density to lower density). Such a generalization is practical because the potential density σ_θ (the density at a given potential temperature is essentially equal to σ_T, which is determined routinely, particularly for the upper 1000 meters. *

Tables for calculation of (1) θ are given by Bialek (1966, p.295-301) and (2) Adiabatic cooling of sea water by Crease and Catton (1961)

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FIGURE 6

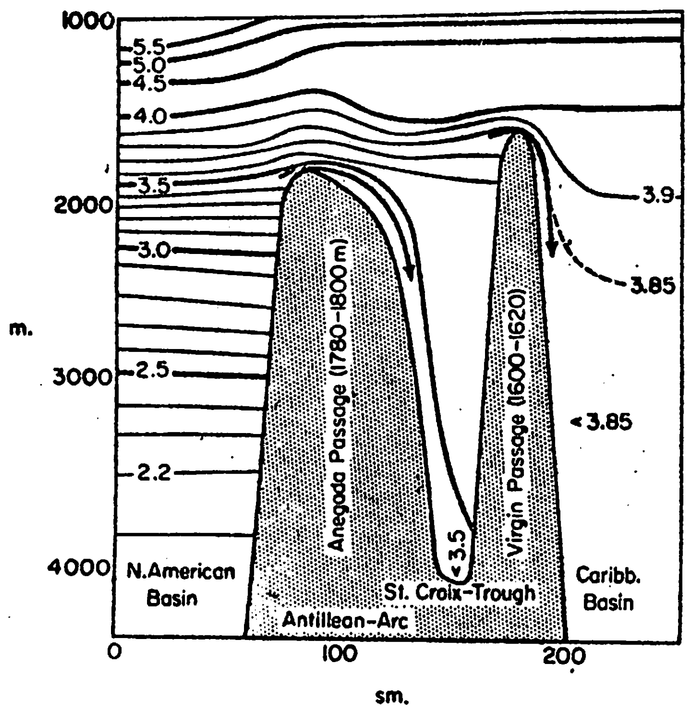

Fig. 207. Vertical distribution of potential temperature (t_p) below 1000 m. along a cross section from the North American Basin through the Anegada Virgin Passage into the Caribbean Basin. Exaggeration of vertical scale 150-fold. (According to G. Dietrich, 1937.)

From Dietrich (1963, p. 499)

In like manner the change in specific volume with pressure for adiabatic conditions, the adiabatic compressibility via EQ (30) is

$$\left(\frac{\partial\alpha}{\partial P}\right)_{N,S} = \left(\frac{\partial\alpha}{T}\right)_{P,S}\left(\frac{\partial T}{\partial P}\right)_{N,S} + \left(\frac{\partial\alpha}{\partial P}\right)_{T,S} \quad (56)$$

or

$$\left(\frac{\partial\alpha}{\partial P}\right)_{N,S} = \Gamma\left(\frac{\partial\alpha}{\partial P}\right)_{P,S} + \left(\frac{\partial\alpha}{\partial P}\right)_{T,S} \quad (57)$$

such a gradient is too small to be of importance for large scale phenomenon such as water mass movements but is important in calculating passage of waves such as sound. For example Beyer (1954) derived the following

$$V = \sqrt{\frac{\alpha^2}{\left(\frac{\partial\alpha}{\partial P}\right)_{N,S}}} \quad (58)$$

as the basic equation for the acoustic velocity in sea water. (see separate section on speed of sound).

Also the effect of pressure on the chemical potential can be treated adiabatically so EQ(35) becomes

$$\left(\frac{\partial\mu}{\partial P}\right)_{N,S} = \left(\frac{\partial\mu}{\partial T}\right)_{P,S}\left(\frac{\partial T}{\partial P}\right)_{N,S} + \left(\frac{\partial\mu}{\partial P}\right)_{T,S} \quad (59)$$

$$= \Gamma\left(\frac{\partial\mu}{\partial T}\right)_{P,S} + \left(\frac{\partial\mu}{\partial P}\right)_{T,S} \quad (60)$$

But substituting EQs (27) and (37).

$$\left(\frac{\partial\mu}{\partial P}\right)_{N,S} = -\Gamma\left(\frac{\partial\overline{N}}{\partial S}\right)_{T,P} + \left(\frac{\partial\alpha}{\partial S}\right)_{T,P} \quad (61)$$

which for adiabatic conditions is

$$\left(\frac{\partial\mu}{\partial P}\right)_{N,S} = \left(\frac{\partial\alpha}{\partial S}\right)_{T,P} \quad (62)$$

or the saline contraction for the particular salinity. Such a function may become more interesting in the future if and when in situ chemical measurements are compared with measurements made at the surface of water brought from depth.

REFERENCES

Beyer, R.T., 1954, Formula For Sound Velocity in Sea Water: Jour. Marine Res., V. 13, pp. 113-121.

Bialek, E.L. (compiler), 1966, Handbook of Oceanographic Tables: U.S. Naval Oceanogr. Off. Spec. Pub. SP-68, 427 p.

Crease, J. and Catton, D., 1961, Tables of Adiabatic Cooling of Sea Water: Nat. Inst. Oceanogr, Godalming, Surrey, England, 7p.

Dietrich, G., 1963, General Oceanography, New York, Interscience, 588p..

Fofonoff, N.P., 1959, Interpretation of Oceanographic Measurements-Thermodynamics: Nat Acad. Sci: N.R.S. Conference on Physical and Chemical Properties of Sea Water, Pub. 600, p. 38-66..

UNIVERSITY OF CALIFORNIA
CE 201A Fall 1971

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Derivation of the Coriolis Acceleration

The earth is a spherical body that rotates about one axis. Thus, to consider the forces acting on the oceans at the surface of the earth, we must first consider its moving frame of reference. To us on earth, the sky rotates east to west. However, it is an apparent motion for the heavenly bodies such as the "fixed" stars are for our purposes stationary. For mathematical convenience the co-ordinate system used here is that of a rotating disk not a sphere, with the center of the fixed or internal frame along the axis of rotation. Thus all sections are planes parallel to the equator. The Z direction will be parallel to the axis of rotation. Therefore X and Y will be perpendicular to the axis of rotation. To us on earth, the apparent x and y axes appear fixed but it is obvious that with respect to the fixed stars, they rotate about Z axis. Thus to investigate motion on the earth to some fixed reference system as the familiar laws of physics usually relate to some fixed reference system. For example

$$F = Ma$$

or more rigorously

$$F = \frac{d[\text{momentum } Mv]}{dt}$$

is true for $F = M a_I$, where the acceleration is measured in an inertial or non-moving frame. But as stated above, the earth is a non-inertial or rotating reference frame. Here we can apply the principle of Galilean invariance which states: the basic laws of physics are unchanged in form in two reference frames connected by a Galilean transform. Thus

$$a_I = a_R + a_o$$

where:

a_R = acceleration with respect to a non-inertial frame (what we would measure on earth)

a_o = acceleration with respect to an inertial frame

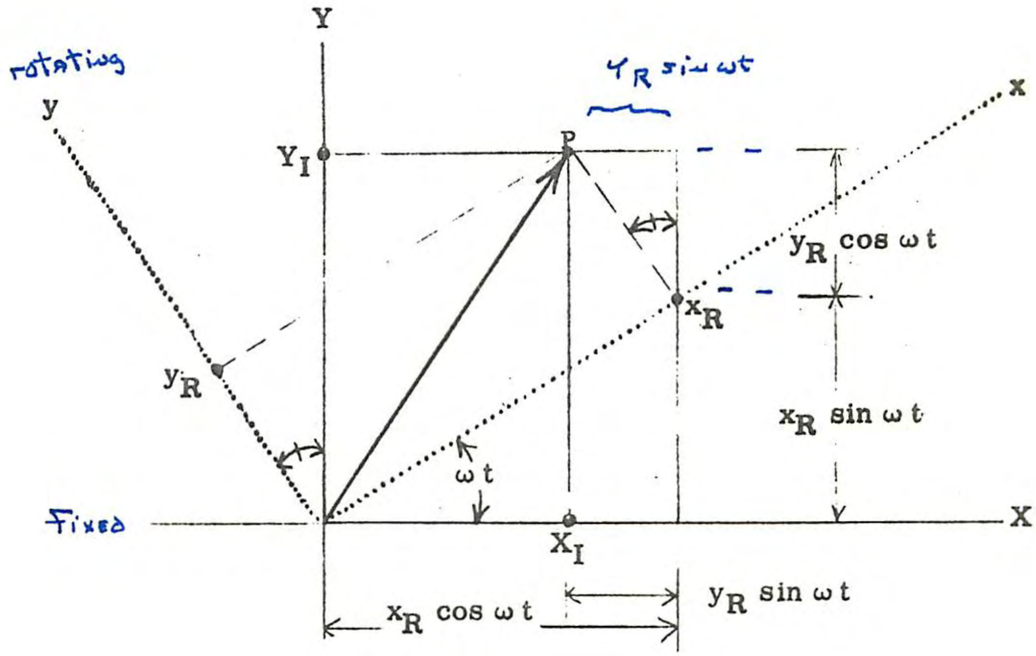
so

$$F = M a_R + M a_o$$

if we define $F_o \equiv - M a_o$. The $F + F_o = M a_R$ where F equals the true force and F_o equals a fictional force due to the non-inertial nature of the earth as a reference plane.

Figure 1 shows a two-dimensional analysis of the earth's rotating system. As the earth only rotates about one axis, the defined Z direction or axis of rotation in both the inertial and rotating reference systems is the same. Thus only the x and y axes, perpendicular to the axis of rotation or the fixed Z in the inertial frame, appear to rotate.

FIGURE 1



Key: (1) Solid lines in the inertial system, (2) dotted lines in the rotation system, (3) point P in inertial co-ordinates $P(X_I, Y_I)$, and (4) point P in rotational co-ordinates $P(x_R, y_R)$.

From Figure 1 for two dimensions

$$X_I = x_R \cos \omega t - y_R \sin \omega t = r_{R(X)}{}^{*}$$

$$Y_I = x_R \sin \omega t + y_R \cos \omega t = r_{R(Y)}{}^{**}$$

which are the Galilean transforms of point P. For three dimensions also add

$$Z_I = z_R$$

---

*The projection of radius r on X.

**The projection of radius r on Y.

The first derivatives of these transforms with respect to time or the velocities are:

$$\frac{d\,X_I}{d\,t} = \dot{X}_I = \dot{x}_R \cos\omega t - x_R\,\omega \sin\omega t - \dot{y}_R \sin\omega t - y_R\,\omega\cos\omega t$$

$$\dot{X}_I = \dot{x}_R \cos\omega t - \dot{y}_R \sin\omega t - \omega\,(x_R \sin\omega t + y_R \cos\omega t)$$

$$\dot{X}_I = V_{R(X)} - \omega\,(r_{R(Y)})$$

and

$$\frac{d\,Y_I}{d\,t} = \dot{Y}_I = \dot{x}_R \sin\omega t + x_R\,\omega\cos\omega t + \dot{y}_R \cos\omega t - y_R\,\omega\sin\omega t$$

$$\dot{Y}_I = \dot{x}_R \sin\omega t + \dot{y}_R \cos\omega t + \omega\,(x_R \cos\omega t - y_R \sin\omega t)$$

$$\dot{Y}_I = V_{R(Y)} + \omega\,(r_{R(X)})$$

and

$$\frac{d\,Z_I}{d\,t} = \dot{Z}_I = \dot{z}_R = v_{R(Z)}$$

The second derivative with respect to time or the accelerations of the transform equations are:

$$\ddot{X}_I = \ddot{x}_R \cos\omega t - \dot{x}_R\,\omega\sin\omega t - \dot{x}_R\,\omega\sin\omega t - \omega^2 x_R \cos\omega t$$

$$- \ddot{y}_R \sin\omega t - \dot{y}_R\,\omega\cos\omega t - \dot{y}_R\,\omega\cos\omega t + \omega^2 y_R \sin\omega t$$

$$= \ddot{x}_R \cos\omega t - \ddot{y}_R \sin\omega t - 2\,\omega\,(\dot{x}_R \sin\omega t + \dot{y}_R \cos\omega t)$$

$$- \omega^2\,(x_R \cos\omega t - y_R \sin\omega t)$$

$$= a_{R(X)} - 2\,\omega\,(v_{R(Y)}) - \omega^2\,(r_{R(X)})$$

11

$$\ddot{Y}_I = \ddot{x}_R \sin \omega t + \dot{x}_R \omega \cos \omega t + \dot{x}_R \omega \cos \omega t - \omega^2 x_R \sin \omega t$$

$$+ \ddot{y}_R \cos \omega t - \dot{y}_R \omega \sin \omega t - \dot{y}_R \omega \sin \omega t - \omega^2 y_R \cos \omega t$$

$$= \ddot{x}_R \sin \omega t + \ddot{y}_R \cos \omega t + 2 \omega (\dot{x}_R \cos \omega t - \dot{y}_R \sin \omega t)$$

$$- \omega^2 (x_R \sin \omega t + y_R \cos \omega t)$$

$$= a_{R(Y)} + 2 \omega (v_{R(X)}) - \omega^2 (r_{R(Y)})$$

and

$$\ddot{Z}_I = \ddot{z}_R = a_{R(Z)}$$

In vector notation:

$$\vec{a}_I = \ddot{X} \hat{X} + \ddot{Y} \hat{Y} + \ddot{Z} \hat{Z}$$

By substitution

$$\vec{a}_I = a_{R(X)} \hat{X} + a_{R(Y)} \hat{Y} + a_{R(Z)} \hat{Z}$$

$$- [2\omega (v_{R(Y)})] \hat{X} + [2 \omega (v_{R(X)})] \hat{Y}$$

$$- [\omega^2 (r_{R(X)})] \hat{X} - [\omega^2 (r_{R(Y)})] \hat{Y}$$

$$= \vec{a}_R + \left[2 \omega [(- v_{R(Y)}) \hat{X} + v_{R(X)} \hat{Y}] \right]$$

$$+ \left[- \omega^2 [(r_{R(X)}) \hat{X} + r_{R(Y)} \hat{Y}] \right]$$

These five vectors can be reduced to just three rotating only on one axis

$$2(\vec{\omega} \times \vec{v}_R) = 2 \begin{vmatrix} \hat{X} & \hat{Y} & \hat{Z} \\ 0 & 0 & \omega \\ v_X & v_Y & v_Z \end{vmatrix} = 2(-\omega v_{R(Y)} \hat{X} + \omega v_{R(X)} \hat{Y}) = 2\omega(-v_{R(Y)} \hat{X} + v_{R(X)} \hat{Y})$$

and the vector

$$\vec{\omega} \times (\vec{\omega} \times \vec{r}_R) = \vec{\omega} \times \begin{vmatrix} \hat{X} & \hat{Y} & \hat{Z} \\ 0 & 0 & \omega \\ r_X & r_Y & r_Z \end{vmatrix}$$

$$= \vec{\omega} \times (-\omega r_{(Y)} \hat{X} + \omega r_{(X)} \hat{Y})$$

$$= \begin{vmatrix} \hat{X} & \hat{Y} & \hat{Z} \\ 0 & 0 & \omega \\ -\omega r_Y & +\omega r_X & 0 \end{vmatrix}$$

$$= -\omega^2 r_{(Y)} \hat{Y} - \omega^2 r_{(X)} \hat{X}$$

$$= -\omega^2 (r_{(Y)} \hat{Y} + r_{(X)} \hat{X})$$

Substituting these new vectors back into the initial vector equation

$$\vec{a}_I = \vec{a}_R + 2(\vec{\omega} \times \vec{v}_R) + \vec{\omega} \times (\vec{\omega} \times \vec{r}_R)$$

Another way to consider these accelerations is from the point of view of the observer on earth

inertial Coriolis Centrifugal

$$\vec{a}_R = \vec{a}_I - 2(\vec{\omega} \times \vec{v}_R) - (\vec{\omega} \times (\vec{\omega} \times \vec{r}_R))$$

where $-2(\vec{\omega} \times \vec{v}_R)$ is designated the deflecting acceleration vector. As can be seen by applying the right hand rule, this vector acts at right angles to the applied velocity vector $\vec{v}_R$, apparently deflecting it. More commonly, this vector is called the Coriolis acceleration.

Thus

$$- M \left| 2 (\vec{\omega} \times \vec{v}_R) \right|$$ ⇐ Coriolis Force

is the coriolis force. Further application of the right hand rule shows that the vector

$$- (\vec{\omega} \times (\vec{\omega} \times \vec{r}_R))$$

is always directed away from the axis of rotation (Remember the negative sign changes the direction of the positive vector 180°.) so is called the centrifugal acceleration and thus

$$- M \left| \vec{\omega} \times (\vec{\omega} \times \vec{r}_R) \right|$$

is the centrifugal force.

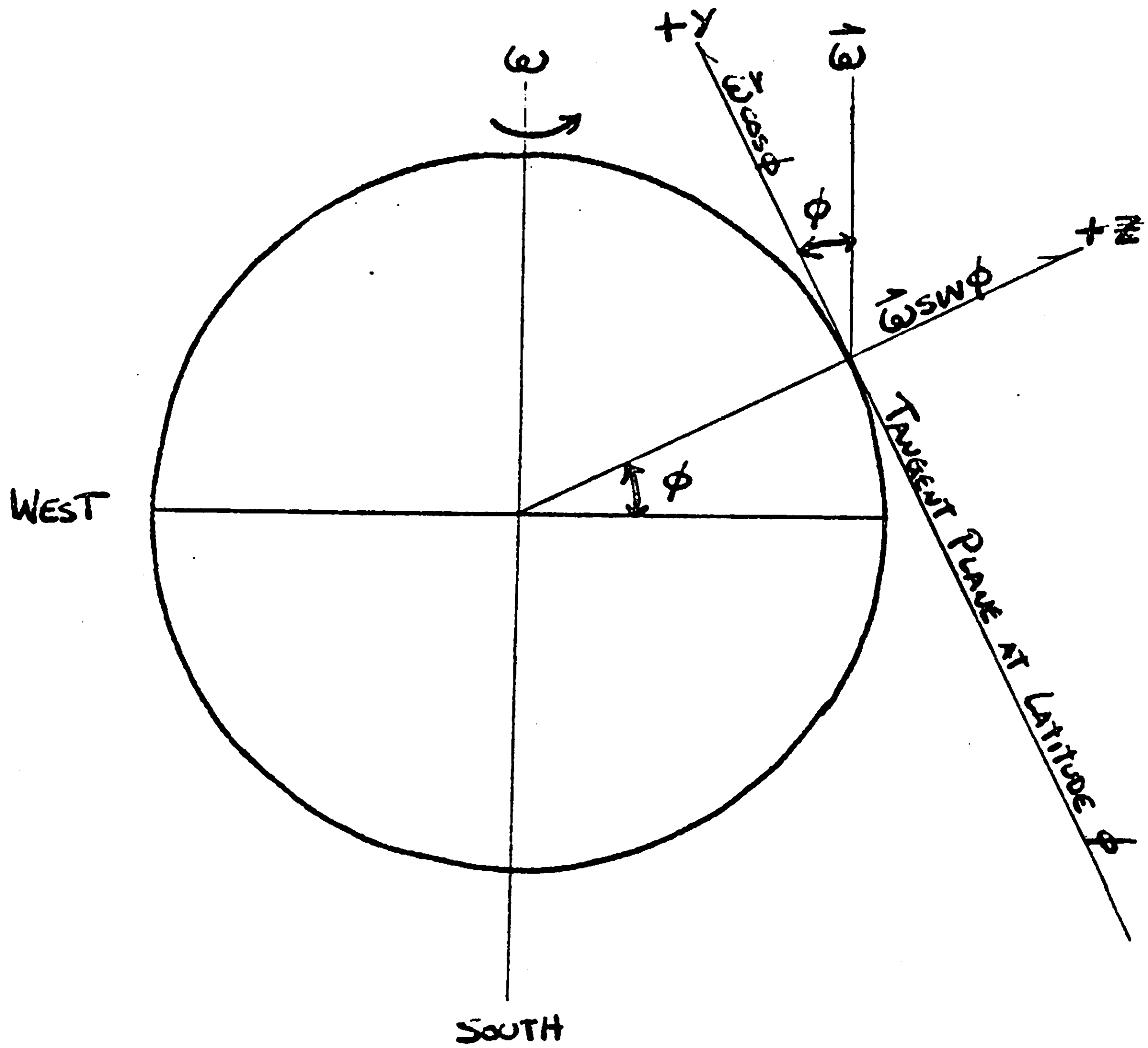
EVALUATION OF THE CORIOLIS ACCELERATION

A. Vector Method

By inspection it is obvious that the direction of the coriolis acceleration vector $- 2 (\vec{\omega} \times \vec{v}_R)$ is determined by the orientation of $\vec{v}_R$, the velocity vector of the moving body with respect to the earth, as $\vec{\omega}$, the axis of rotation of the earth, is fixed. In oceanography the moving bodies of interest are ocean currents, which for our purposes move much faster in the horizontal direction than in the vertical plane. Therefore it would be convenient to know the direction of the coriolis or deflecting acceleration with the same certainty to which we know the direction of the centrifugal acceleration

The preceeding discussion was based on accelerations on planes perpendicular to the axis of rotation. However, the practical plane for earth dwellers is the tangent plane to the earth's surface. For any tangent plane to the earth's surface let X be positive East, and Y be positive North and the normal to the plane Z be positive up. $\vec{\omega}$ also has components in the Y and Z directions but as the earth rotates west to east $\vec{\omega} = 0$ along the X or East-West direction. Thus the determinant for the Coriolis acceleration for the tangent place is:

$$-2\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & \omega\cos\emptyset & \omega\sin\emptyset \\ V_x & V_y & V_z \end{vmatrix} = \begin{array}{ll} -2\omega\cos\emptyset\, V_z\, \hat{x} & \text{(a)} \\ -2\omega\sin\emptyset\, V_x\, \hat{y} & \text{(b)} \\ +2\omega\cos\emptyset\, V_x\, \hat{z} & \text{(c)} \\ +2\omega\sin\emptyset\, V_y\, \hat{x} & \text{(d)} \end{array}$$



Where (a) $-2\cos\phi\ V_z\ \hat{x}$ is the horizontal deflection on a vertical velocity

(b) $-2\omega\sin\phi\ V_x\ \hat{Y}$ and (d) $+2\omega\sin\phi\ V_y\ \hat{X}$ are horizontal deflections on horizontal velocities and

(c) $+2\omega\cos\phi\ V_x\ \hat{Z}$ is the vertical deflection on a horizontal velocity.

✱

The absolute value of the Coriolis deflection is $2|\omega||V_r|\sin\phi$ where ω is the angle between $\vec{\omega}$ and $\vec{V}_r$ or (1) $2|\omega||V_r|\sin\phi$ for deflections in the horizontal plane and (2) $2|\omega||V_r|\cos\phi$ for deflections in the vertical plane. 2ω (ω is the angular velocity of the earth's rotation) has a constant value of about 1.5×10^{-4}/sec. Accordingly the Coriolis deflection is rather small, the maximum value being only 1.5×10^{-4}/sec times the initial velocity V_r.

$\begin{cases} \phi + & \text{N.H} \\ \phi - & \text{S.H} \end{cases}$

With the additional convention that the geographic latitude ϕ is positive in the northern hemisphere and is negative in the southern hemisphere these four vectors give the orientation as well as the magnitude of the Coriolis deflection for the tangent plane and its normal to the earth's surface. ✱

X Direction

EXAMPLE A current flowing East at 10cm/sec at 50°N. (b) $-2\omega\sin\phi\ V_x\ \hat{Y}$ and (a) $+2\omega\cos\phi\ V_x\ \hat{z}$ obtain. Thus (b) Horizontal deflection South (-y); magnitude: $2\omega\sin 50^{\circ}$ (10cm/sec) and (c) vertical deflection up (+Z); magnitude: $2\omega\cos50^{\circ}$ (10cm/sec).

Y Direction

EXAMPLE A current flowing North at 25cm/sec at 15°S. (d) $+2\omega\sin\phi\ V_y\ \hat{X}$ obtains as ϕ negative $-2\omega\sin\phi\ V_y\ \hat{X}$. Thus (d) Horizontal deflection West (-X); magnitude: $2\omega\sin15^{\circ}$ (25cm/sec).

or as follows:

Term	Orientation of V	Orientation of Coriolis Deflection Northern Hemisphere	Southern Hemisphere
$-2\omega\cos\phi V r_z \hat{X}$	up	West	West
	down	East	East
$-2\omega\sin\phi V r_x \hat{Y}$	East	South	North
	West	North	South
$+2\omega\cos\phi V r_x \hat{Z}$	East	up	up
	West	Down	Down
$+2\omega\sin\phi V r_y \hat{X}$	North	East	West
	South	West	East

These sets of deflections can be remembered as (1) to the right in the North Hemisphere and (2) to the left in the Southern Hemisphere for an observer (a) facing in the direction of motion and either, (b) standing upright for horizontal deflections or (c) lying down with his head pointed toward the pole for vertical deflections.

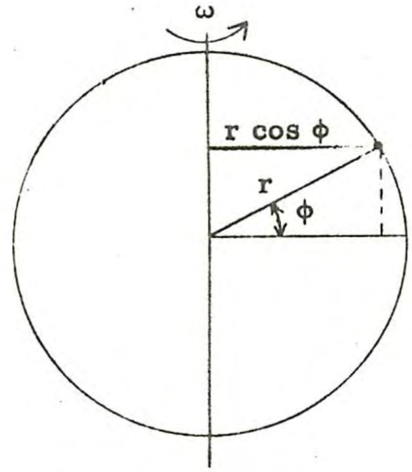
Using the solutions of the determinate to figure the orientation of the Coriolis acceleration eliminates possible confusion in the use of the right hand rule for negative cross products such as $-2(\vec{\omega} \times \vec{V})$.

$\sin\phi$ is at a maximum value = 1, at the poles 90° N. or 90° S. and zero at the equator. However $\cos\phi$ is zero at the poles and = 1 at the equator. Thus horizontal vectors show maximum horizontal Coriolis deflection at the equator. ?, On the other hand a vertical vector shows maximum deflection at the equator and no deflection at the poles.

Angular Momentum Method

Because the earth rotates about one axis, the angular momentum of a particle at the surface varies with the cosine of latitude or is equal to $M \omega r \cos\phi$ where: ω = the angular velocity; r = radius; ϕ = geographic latitude; M = mass of particle

CHANGE IN ANGULAR MOMENTUM OF PARTICLE AT REST WITH LATITUDE



Thus, the absolute velocity of a particle at the equator at rest is higher than for a particle at rest elsewhere.

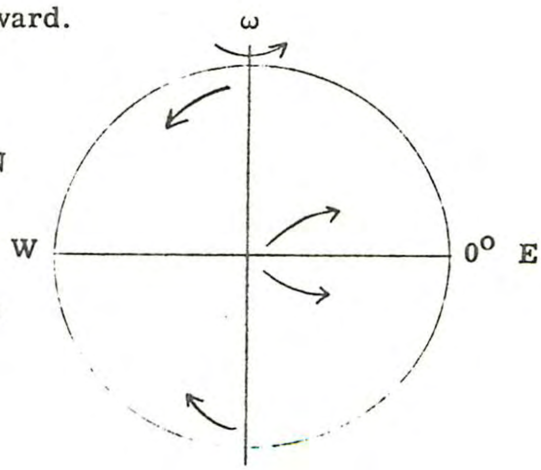
Velocity in Miles per Hour	Latitude
1,000	0^{o}
866	$\pm\ 30^{o}$
500	$\pm\ 60^{o}$
0	$\pm\ 90^{o}$

As a particle moves to higher latitudes it has a higher angular momentum than a particle at rest. As a consequence, the moving particle appears to move faster than the earth, or to an observer at rest, the moving particle appears to be deflected eastward.

On the other hand if the particle moves from a higher to a lower latitude, the moving particle appears to lag behind the earth's rotation, or to an observer, the moving particle is deflected westward.

APPARENT DEFLECTION OF MOVING PARTICLE

(Observer facing direction of pointing arrow)



Thus to an observer at the earth's surface

A. In the Northern Hemisphere, a moving particle appears to be deflected to the right of its direction of movement, and

B. In the Southern Hemisphere, a moving particle appears to be deflected to the left of its direction of movement.

As noted in the previous section and shown by the determinate vectors, the horizontal Coriolis deflection is to the right (facing in the direction of the velocity vector) in the Northern Hemisphere and to the left in the Southern Hemisphere, to eliminate this combersome designation of hemisphere oceanographers use the phrase <u>cum sole</u> (Latin: with the sun) to designate the orientation of the horizontal Coriolis deflection on a horizontal vector; as the sun from dawn to dusk moves to the right in the Northern Hemisphere and to the left in the Southern Hemisphere with respect to an observer in that hemisphere.

Opposite motions, accordingly, are designated <u>contra solem</u> (Latin: against the sun).

<u>References</u>

Kittel, C., Knight, W.D., and Ruderman, M.A., 1965 <u>Mechanics</u>; Berkeley Physics Course: McGraw Hill, New York, 480 p.

Von Arx, W. S., 1962, <u>Introduction to Physical Oceanography</u>: Addison-Wesley, R-ading, Mass., 422p. (p. 87-88 on conservation of angular momentum.)

OCEAN ENGINEERING

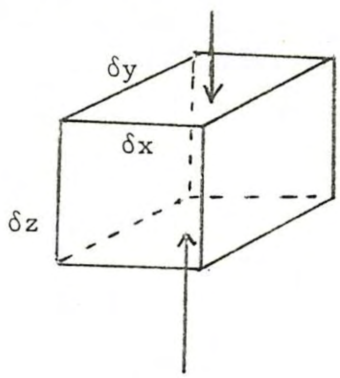
C.E. 201A

PHYSICAL OCEANOLOGY

DERIVATION OF THE GEOSTROPHIC EQUATION FOR THE OCEANS

Hydrostatic conditions:

Given a cube of sea water -



The pressure on the top of the cube or force/unit area

$$P_T = \frac{ma}{\delta x\ \delta y}$$

The pressure on the bottom of the cube

$$P_B = P_T + \delta P.$$

If we take vertically downward z as positive, then the force on the top -

$$F_T = P_T\ (\delta x\ \delta y)$$

and the force on the bottom:

$$F_B = -(P_T + \delta P)\ \delta x\ \delta y$$

The net force: $\delta F = -\delta P(\delta x\ \delta y)$

as $\frac{\text{force}}{\text{unit mass}}$ = acceleration

and $m = \rho \ell^3 = \rho(\delta x\ \delta y\ \delta z)$

the acceleration $a = -\frac{1}{\rho}\ \frac{\delta P(\delta x\ \delta y)}{(\delta x\ \delta y\ \delta z)}$

$$= -\frac{1}{\rho}\ \frac{dP}{dz}$$

directed upward.

If the cube is stationary, the acceleration $-\frac{1}{\rho}\frac{dP}{dz}$ must be balanced against g, which is acting vertically down or

$$-\frac{1}{\rho}\frac{dP}{dz} + g = 0 \qquad \text{or}$$

$$dP = \rho g dz$$

In three dimensions the resultant pressure gradient can be represented vectorially by the grad or

$$\nabla P = \hat{\imath}\frac{\partial P}{\partial x} + \hat{\jmath}\frac{\partial P}{\partial y} + \hat{k}\frac{\partial P}{\partial z}$$

which is directed normal to the isobaric (equal pressure) surfaces. In the oceans pressure increases with depth so that the pressure gradient ∇P, which is directed from high to low pressure, is always negative in the oceanic co-ordinate system as the positive z direction is vertically downward. *

Case I. Isobars parallel with geopotential surfaces

$$\begin{array}{ll} \rule{8cm}{0.4pt}\ P_1 & P_3 > P_2 > P_1 \\ \rule{8cm}{0.4pt}\ P_2 & \frac{\partial P}{\partial x} = \frac{\partial P}{\partial y} = 0 \\ \rule{8cm}{0.4pt}\ P_3 & \nabla P = -\hat{k}\frac{\partial P}{\partial z} \end{array}$$

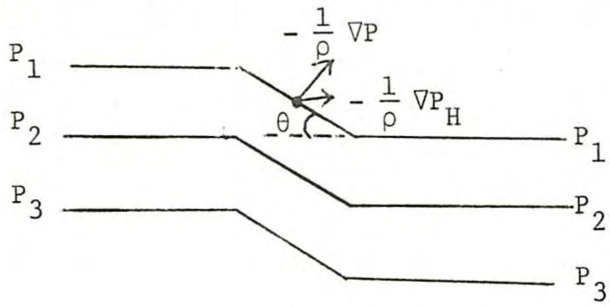
The acceleration caused by the pressure gradient:

$$\hat{a}_P = -\frac{1}{\rho}\hat{k}\frac{\partial P}{\partial z}$$

The acceleration in the horizontal plane

$$\hat{a}_H = 0$$

Case II. Isobars inclined to geopotential surfaces

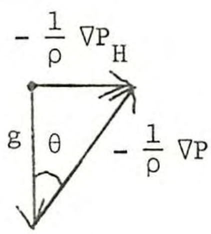


$$P_3 > P_2 > P_1$$

$$\frac{\partial P}{\partial x} \neq 0; \quad \frac{\partial P}{\partial y} \neq 0$$

$$\nabla P = -\hat{i}\,\frac{\partial P}{\partial x} \; -\hat{j}\,\frac{\partial P}{\partial y} \; -\hat{k}\,\frac{\partial P}{\partial z} \Big\}$$

$$a_P = -\frac{1}{\rho}\,\hat{i}\,\frac{\partial P}{\partial x} - \frac{1}{\rho}\,\hat{j}\,\frac{\partial P}{\partial y} - \frac{1}{\rho}\,\hat{k}\,\frac{\partial P}{\partial z}$$



With inclined isobars the pressure gradient has horizontal components so

$$\hat{a}_H = \hat{g}\tan\theta = -\frac{1}{\rho}\,\hat{i}\,\frac{\partial P}{\partial x} - \frac{1}{\rho}\,\hat{j}\,\frac{\partial P}{\partial y}$$

$$= -\frac{1}{\rho}\,\nabla P_H$$

(upward ⟹ − sign in oceanog. ref. frame)

Usually in oceanographic equations $\frac{1}{\rho}$ is replaced by the specific volume α where

$$\alpha = \frac{1}{\rho} \qquad \text{thus}$$

$$\hat{a}_H = -\alpha\,\nabla P_H$$

EQUATIONS OF MOTION

We are now able to describe the accelerations on a particle of sea water on a rotating earth. To recapitulate -

$$\hat{a}_I = \hat{a}_R + \hat{a}_o$$

where $\hat{a}_o = 2(\hat{\omega} \times \hat{V}_R) + \hat{\omega} \times (\hat{\omega} \times \hat{R}_R)$. With respect to an observer on earth

$$\hat{a}_R = \hat{a}_I - \hat{a}_o$$

Adding the accelerations due to gravity, the pressure gradient, and friction we have

$$\hat{a}_R = \hat{a}_I + (-2\ (\hat{\omega} \times \hat{V}_R)) + (-\hat{\omega} \times (\hat{\omega} \times \hat{R}_R)) + (-\alpha\ \nabla P) + \hat{g} + \hat{F}$$

$|\hat{a}_I|$ is small except for velocities in excess of the tangential speed of the earth. $|(-\hat{\omega} \times (\hat{\omega} \times \hat{R}_R)|$ the centrifugal acceleration is small compared to g. So for practical oceanic measurements

$$a_R = \frac{d\ V_R}{dt} = \text{the coriolis acceleration}$$

+ the pressure gradient acceleration

+ the acceleration due to gravity

+ the accelerations caused by friction

*

For an oceanic co-ordinate system:

x positive towards the east

y positive towards the north

z positive vertically downward

V_R can be resolved into components u, v, and w along the x, y, and z axes respectively.

The coriolis acceleration vector is

$$-\ 2(\hat{\omega} \times \hat{V}_R) =$$

$$-2 \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & \omega\cos\phi & \omega\sin\phi \\ V_x & V_y & V_z \end{vmatrix} = \begin{array}{l} -2\ \omega\cos\phi\ V_z\ \hat{x} \\ -2\ \omega\sin\phi\ V_x\ \hat{y} \\ +2\ \omega\cos\phi\ V_x\ \hat{z} \\ +2\ \omega\sin\phi\ V_y\ \hat{x} \end{array}$$

For the horizontal and vertical planes replacing V_x by u; V_y by v; and V_z by w, then

$$\frac{du}{dt} = 2\,\omega \sin\phi\, v - \alpha \frac{\partial P}{\partial x} + F_x$$

$$\frac{dv}{dt} = -2\,\omega \sin\phi\, u - \alpha \frac{\partial P}{\partial y} + F_y$$

$$\frac{dw}{dt} = + g - \alpha \frac{\partial P}{\partial z} + F_z$$

These equations describe the Eulerian form or defines the motion of particles at a particular instant of time. The Eulerian form is suitable for at-a-station data such as that derived from a Nansen bottle cast. An alternate form is the Lagrangian which follows the path of one particle through time. The Lagrangian form would be appropriate for studies of drift stations such as swallow buoys. The Eulerian form can be expanded to:

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$\frac{dw}{dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

As the measurements are taken at a fixed point in the co-ordinate system the particles themselves move past the fixed point. So the accelerations must be resolved into fixed (local) and moving (advective) parts if the motion is to be studied for longer than an instant.

If the local derivatives vanish the motion is stationary--no change at a given point. As the oceans and the atmosphere are already in motion and appear to have been for long periods it is not necessary to worry about starting circulation and it is much easier to investigate steady

(Lag. analysis is rare)

state conditions or conditions where flow exists without time dependent changes.

If the local derivatives

$$\frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = \frac{\partial w}{\partial t} = 0$$

a steady state exists and if both the local and advective terms = 0, the fluid is said to be in equilibrium, or

$$\frac{du}{dt} = 0 = 2\ \omega \sin\phi\ v - \alpha \frac{\partial P}{\partial x} + F_x$$

$$\frac{dv}{dt} = 0 = -2\ \omega \sin\phi\ u - \alpha \frac{\partial P}{\partial y} + F_y$$

$$\frac{dw}{dt} = 0 = g - \alpha \frac{\partial P}{\partial z} + F_z$$

In general friction is a problem at boundary layers where the difference in density between the layers is large as at the ocean surface between sea water and air and at the ocean floor between sea water and bottom sediments or rock. In open ocean water away from regions of effective boundary friction, unaccelerated and frictionless flow is called GEOSTROPHIC (earth turned) and the following are the geostrophic equations:

$$\frac{1}{\rho}\ \frac{\partial P}{\partial x} = 2\ \omega \sin\phi\ v \qquad \text{(in the x direction)}$$

$$\frac{1}{\rho}\ \frac{\partial P}{\partial y} = -2\ \omega \sin\phi\ u \qquad \text{(in the y direction)}$$

$$\frac{1}{\rho}\ \frac{\partial P}{\partial z} = g \qquad \text{(in the z direction)}$$

where the only effective forces per unit mass acting are (1) the pressure gradient, (2) the coriolis, and (3) gravity. To simplify writing these equations, let $f = 2\ \omega \sin\phi$ which is constant at any given latitude and

$$C = \sqrt{u^2 + v^2} \quad \text{and}$$

$$\frac{\partial P}{\partial N} = \sqrt{\left(\frac{\partial P}{\partial x}\right)^2 + \left(\frac{\partial P}{\partial y}\right)^2}$$

Geostrophic Equation $\alpha \nabla P_h = f c$

" Velocity $c = \dfrac{\alpha \nabla P_h}{f}$

where ∂P = a measured pressure difference along a line in the xy plane

∂N = measured along line of maximum P

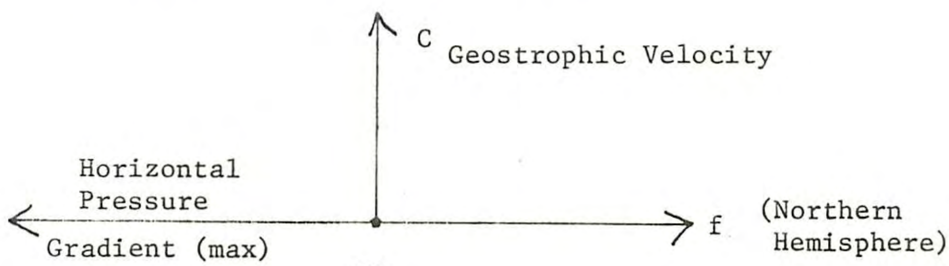
Thus, in the horizontal plane

$$\frac{1}{\rho}\ \frac{\partial P}{\partial N} = \underbrace{2\ \omega \sin\phi}_{f}\ C$$

$$\alpha\ \frac{\partial P}{\partial N} = fC \qquad \text{where}$$

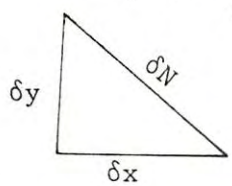
C = the geostrophic velocity.

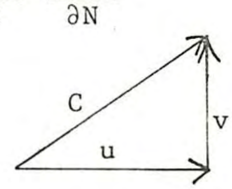
The relationships between the terms are as follows:



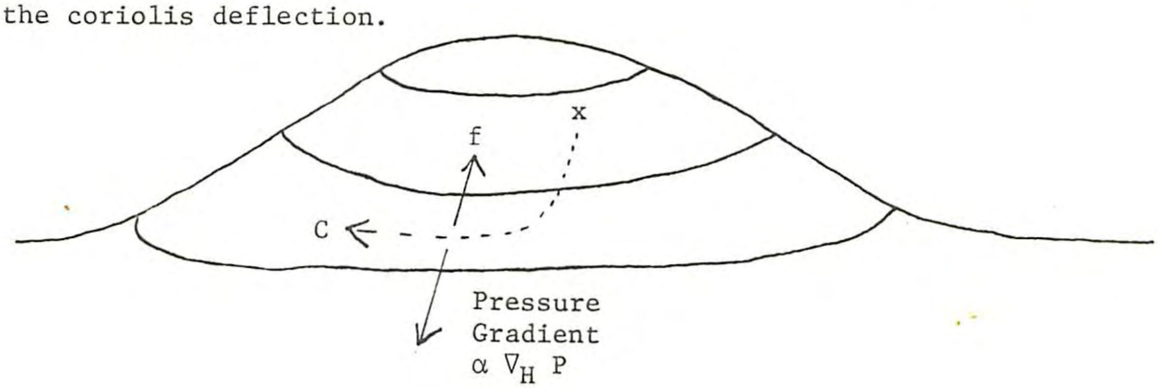
or

$$C = \frac{1}{\rho f}\ \frac{\partial P}{\partial N} = \frac{\alpha}{f}\ \frac{\partial P}{\partial N}$$

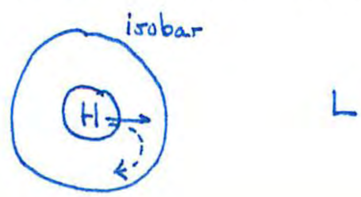




Thus, C is directed $\perp$ to both the maximum horizontal pressure gradient and the coriolis deflection.



Given a pressure field in the horizontal plane which is represented above as analogous to a geopotential contoured surface, isobars analogous to



geostrophic velocity is when the geostrophic balance is attained

geopotential isolines. The pressure gradient produces a horizontal force along δN in the direction of lower pressure. Due to the rotation of the earth the motion of particles (in the northern hemisphere) will be deflected to the right. In a frictionless geostrophic condition the particle will eventually take a path parallel to the isobars. The geopotential analogy would be that of a stone rolling down hill (to a lower geopotential isoline) gradually being deflected such that the force produced by the geopotential gradient would be balanced by the deflecting force such that the particle moves in a path parallel to the geopotential surfaces.* This is exactly the geostrophic assumption, that is, the horizontal pressure gradient is balanced against the coriolis deflection or *

GEOSTROPHIC BALANCE

$$fC - \alpha \frac{\partial P}{\partial N} = 0$$

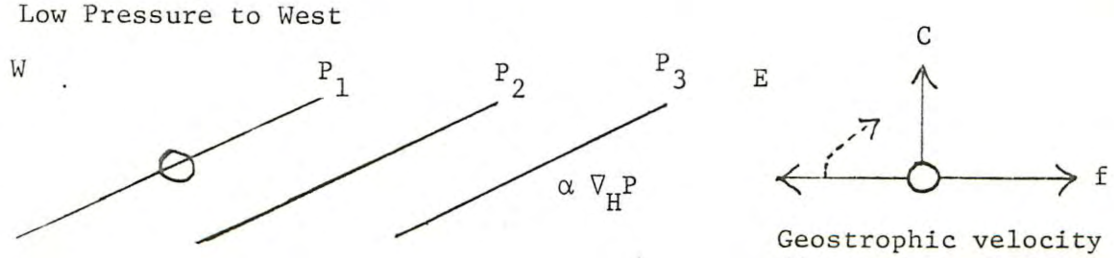
$$fC = \alpha \frac{\partial P}{\partial N}$$

The geostrophic assumption permits the computation of a vector field of motion from the scalar field of pressure.

It is possible to predict the direction of the geostrophic velocity from knowledge of (1) the direction of slope of the isobars and (2) the hemisphere of the measurement.

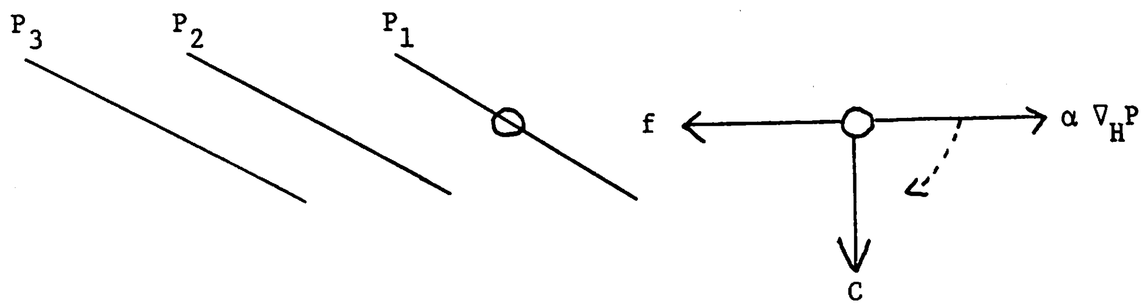
Northern Hemisphere $(P_3 > P_2 > P_1)$

(A) Low Pressure to West



Geostrophic velocity directed north

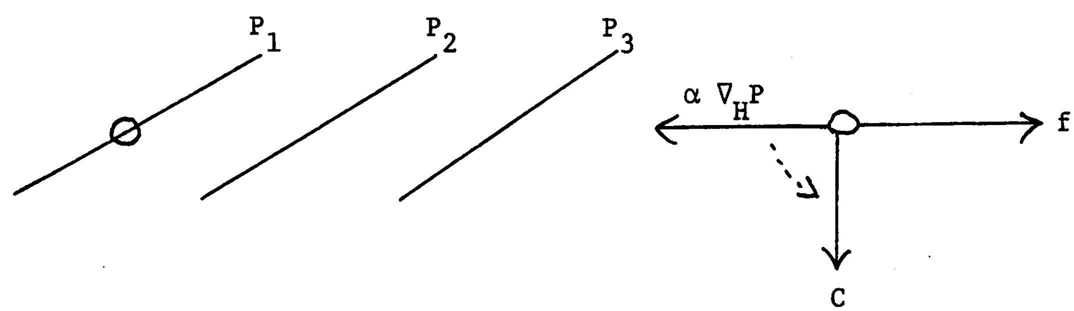
(B) Low Pressure to East



Geostrophic velocity
directed south

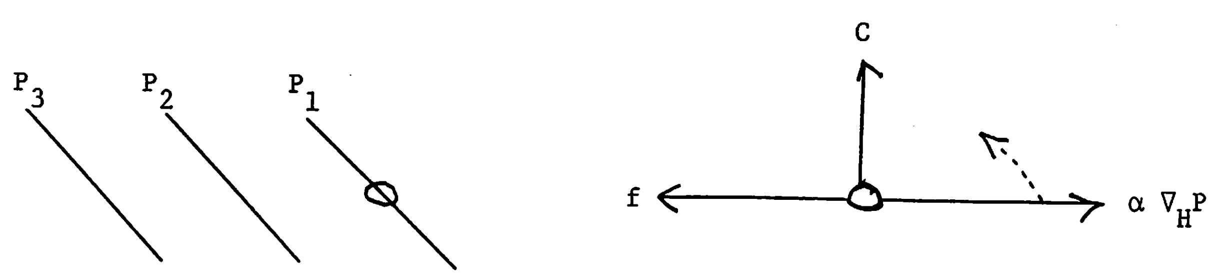
Southern Hemisphere $(P_3 > P_2 > P_1)$

(A) Low Pressure to West



Geostrophic velocity
south

(B) Low Pressure to East



Geostrophic velocity
directed south

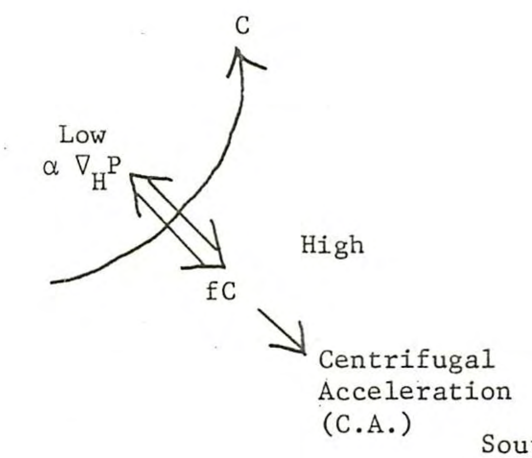
<u>MEANDER MOTION</u>

Where pressure systems are curved or meandering the local centrifugal accelerations must be considered.

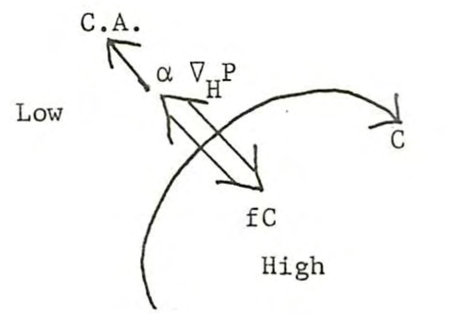
Atmospheric Flow

Northern Hemisphere

Cyclonic Flow

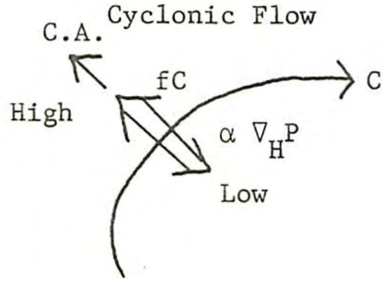


Anticyclonic Flow

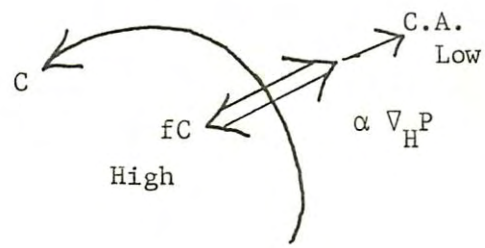


Southern Hemisphere

Cyclonic Flow



Anticyclonic Flow



The diagrams show that in either hemisphere cyclonic flow the centrifugal and coriolis accelerations are added or:

$$\alpha \frac{\partial P}{\partial N} = fC + \frac{C^2}{r} \quad \text{(cyclonic flow)}$$

r = radius of curvature

whereas in anticyclonic flow the horizontal pressure gradient term and the centrifugal acceleration are added or:

$$\alpha \frac{\partial P}{\partial N} = fC - \frac{C^2}{r} \quad \text{(anticyclonic flow)}$$

The quadratic form of anticyclonic flow is:

$$C^2 - rfC + \alpha r \frac{\partial P}{\partial N} = 0$$

$$C = \frac{rf}{2} \pm \frac{1}{2}\sqrt{rf^2 - 4r\alpha \frac{\partial P}{\partial N}}$$

$$C = \frac{rf}{2} \pm \frac{1}{2}\sqrt{r(rf^2 - 4\alpha \frac{\partial P}{\partial N})}$$

If $4\,\alpha \frac{\partial P}{\partial N} \geq rf^2$ the root becomes imaginary. Thus, 4 times the horizontal pressure gradient can never exceed the square of the coriolis parameter f. At the center of a high pressure system where r is small, the pressure gradient must be small so the velocities are small. Thus, high pressure centers are regions of calms.

The quadratic form for cyclonic flow is:

$$C^2 + rfC - r\alpha \frac{\partial P}{\partial N} = 0$$

$$C = -\frac{rf}{2} \pm \frac{1}{2}\sqrt{r(rf^2 + 4\alpha \frac{\partial P}{\partial N})}$$

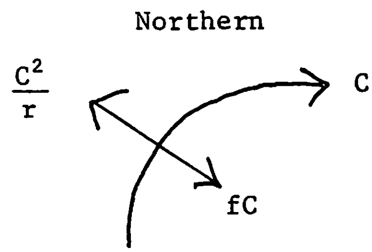
In the case where r is small near the center of a low pressure area, the pressure gradient can exceed the coriolis term as both are added under the radical. All roots are real. This indicates that high velocities can exist at the center of low pressure systems due to the build up of pressure gradients.

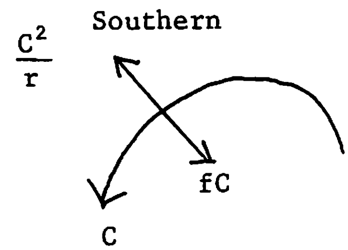
INERTIAL MOTION

Inertial flow occurs when the centrifugal acceleration balances the coriolis acceleration or when there is no horizontal pressure gradient. In this case

$$\frac{C^2}{r} = fC$$

By definition inertial motion is anticyclonic as:





At the same latitude, as f or $2\omega\sin\phi$ is constant, the inertial path is circular. If the particle changes latitude (a more practical case) the radius of curvature will change as f changes because

$$r = \frac{C}{f}$$

So as f increases (the particle moves to a higher latitude) r decreases and the path becomes more tightly curved. As f decreases (the particle moves to lower latitudes) r increases and the inertial path broadens.

The period for one circuit of the inertial circle:

$$T = \frac{2\pi r}{C} = \frac{2\pi r}{2\omega\sin\phi r}$$

$$= \frac{\pi}{\omega\sin\phi}$$

$\frac{\pi}{\omega}$ = 12 sideral hours

So $T = \frac{12}{\sin\phi}$ hours which is called the half pendulum day as this is the period of the Foucault pendulum. Because of the direction of rotation of the earth west to east the path of the inertial particle drifts westward.

UNIVERSITY OF CALIFORNIA
Fall Quarter 1970
CE 201 A

Department of Civil Engineering
Ocean Engineering

USE OF THE GEOSTROPHIC ASSUMPTION

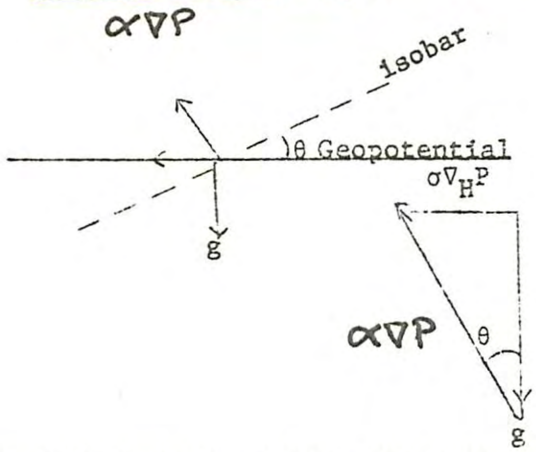
I. GRADIENT CURRENTS

Given non-frictional geostrophic conditions, currents produced as a result of the slope of isobaric surfaces are called gradient currents. As:

$$\alpha \frac{\partial P}{\partial X} = 2\omega \, \text{SIN}\phi v$$

$$\alpha \frac{\partial P}{\partial y} = 2\omega \, \text{SIN}\phi u$$

Assume under constant atmospheric conditions the slope of the sea surface is an isobaric surface.



θ = Slope of sea surface
∇P = Pressure gradient
$\nabla_H P$ = Horizontal component of pressure gradient

$$\text{SO TAN } \theta = \frac{\alpha \nabla_H P}{g}$$

$$\left[g \text{ TAN } \theta = \alpha \nabla_H P \right]$$

For maximum θ or maximum pressure gradient

$$\alpha \nabla_H P = \alpha \frac{\partial P}{\partial N}$$

As the geostrophic equation is

$$fc = 2\omega \, \text{SIN}\phi C = \alpha \frac{\partial P}{\partial N}$$

$$fc = g \tan \theta \text{ max}$$

or $$\tan \theta = \frac{fc}{g} \quad \text{(the gradient equation)}$$

thus the geostrophic gradient velocity $$c = \frac{g \tan \theta \text{ max}}{f}$$


P_1
P_2

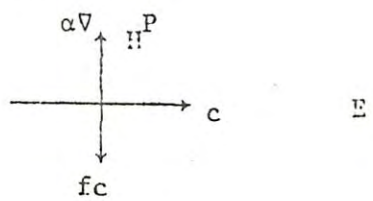
Examples using the gradient equation

A. The equatorial countercurrent flows east at 10°N at 51.4 cm/sec.

$$\tan\ \theta = \frac{fc}{g}$$

$$= \frac{2 \times (7.29 \times 10^{-5}/\text{sec}) \times 1.74 \times 10^{-1} \times (5.14 \times 10^{1}\ \text{cm/sec})}{9.78 \times 10^{2}\ \text{cm/sec}^2}$$

$$= 1.33 \times 10^{-5}$$



B. In straits of Dover non-tidal flow to the north so the slope of the isobar surface is east to west

$\phi = 51°N$ $\qquad$ $c = 6.43$ cm/sec

$$\tan\ \theta = \frac{2 \times (7.29 \times 10^{-5}/\text{sec}) \times (7.771 \times 10^{-1}) \times (6.43 \times 10^{0}\ \text{cm/sec})}{9.81 \times 10^{-2}\ \text{cm/sec}}$$

$$= 7.43 \times 10^{-7}$$

= .743 mm/km Drop from French to English coast

Distance across channel = 35.2 km

Total drop = (35.2 km)(0.743 mm/km)

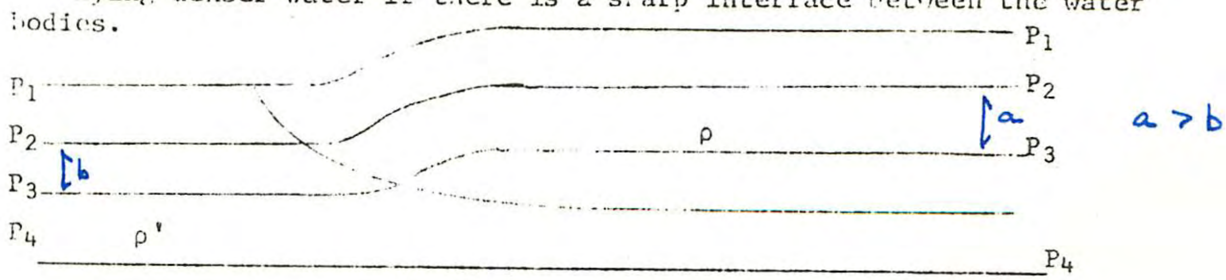
= 26.2 mm

= 2.62 cm

Thus there is a different in mean sea level of 2.62 cm caused by the non-tidal gradient current.

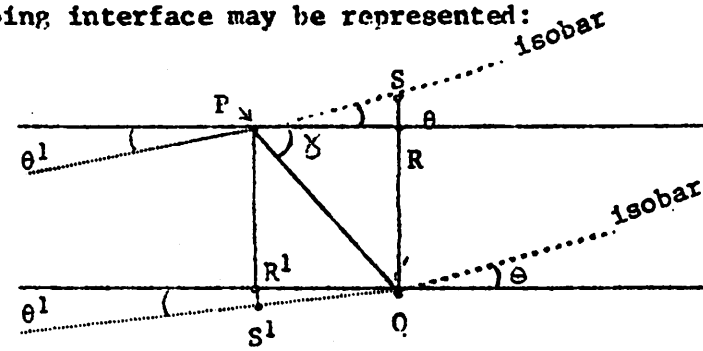
II. SLOPE CURRENTS

There also could be differences in level produced by lighter density water overlying denser water if there is a sharp interface between the water bodies.



for $\rho^1 > \rho$

The sloping interface may be represented:



θ = Slope of isobars in ρ

θ^1 = Slope of isobars in ρ^1

γ = Slope of interface

Increase in pressure from P to S^1 in medium ρ^1 must equal increase in pressure from S to O in medium ρ so:

$$g\rho\, SO = g\rho^1\, PS^1$$

$$\rho(SR + RO) = \rho^1(PR^1 + R^1S^1)$$

$$\tan\theta = \frac{SR}{PR}\ ; \quad \tan\gamma = \frac{RO}{PR} = \frac{PR^1}{R^1O}$$

$$\tan\theta^1 = \frac{R^1S^1}{R^1O} \quad \text{AND} \quad \frac{RO}{PR} = \frac{PR^1}{R^1O}$$

SO:

$$SR = PR\tan\theta$$
$$RO = PR\tan\gamma$$
$$PR^1 = PR\tan\gamma$$
$$R^1S^1 = PR\tan\theta^1$$

and $\rho PR(\tan\theta + \tan\gamma) = \rho^1 PR(\tan\theta^1 + \tan\gamma)$

$$\tan\gamma\,(\rho^1-\rho) = \rho\tan\theta - \rho^1\tan\theta^1 \qquad \tan = \frac{\rho\tan\theta - \rho^1\tan\theta^1}{\rho^1-\rho}$$

as $\tan\theta\ \dfrac{2\omega\,\mathrm{SIN}\phi C}{g} = \dfrac{fc}{g}$

$$\tan\theta^1 = \frac{2\omega\,\mathrm{SIN}\phi C^1}{g} = \frac{fc^1}{g}$$

then $\tan\gamma = \dfrac{f}{g}\,\dfrac{(\rho c-\rho^1c^1)}{(\rho^1-\rho)}$ <u>MARGULES EQUATION</u>

which was initially developed for atmospheric problems.

Example using Margules Equation

A. East Greenland current with $\sigma_T = 27.1$ flows south at 73°N at 20 cm/sec over still water with $\sigma_T = 28.1$

$$\tan \gamma = \frac{f}{g} \; \frac{(\rho c - \rho^1 o)}{\rho^1 - \rho}$$

$$= \left[\frac{2 \times (7.29 \times 10^{-5}/\text{sec})(9.56 \times 10^{-1})}{9.81 \times 10^{2} \text{ cm/sec}^2}\right] \quad X$$

$$\left[\frac{(1.0271) \times (2 \times 10^{1} \text{ cm/sec})}{1.0281 - 1.0271}\right]$$

$$= 2.92 \times 10^{-3}$$

$$= 2.93 \text{ meters/kilometer } \cancel{\times 10^{?}}$$

$$\tan \gamma = \frac{(1.5 \times 10^{-4}/\text{sec})(9.56 \times 10^{-1})(2 \times 10^{1} \text{ cm/sec})}{9.81 \times 10^{2} \text{ cm/sec}^2}$$

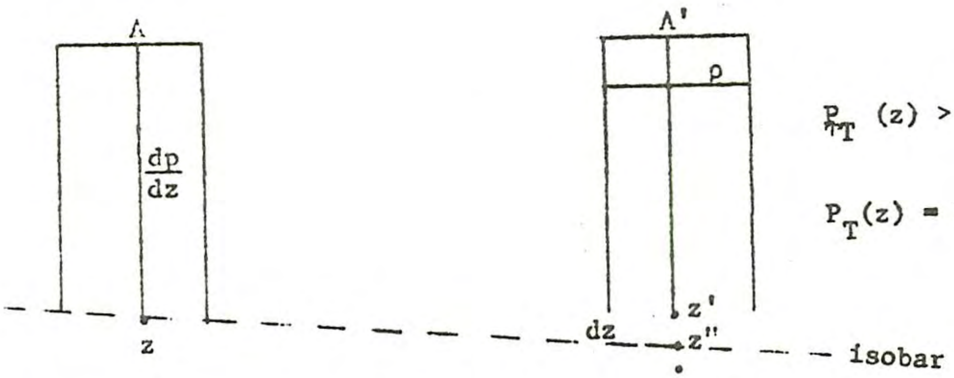
$$= 2.96 \times 10^{-6}$$

The above discussion ignored the fact that the total pressure at a point is composed of two components (1) a relative pressure which is derived from the variations in the field of mass (as considered in the derivation of the gradient equation) and (2) a slope field produced by the pile up of material of the same mass (as indicated by the derivation of the Margules Equation.) The relative gradient current which is produced by the relative pressure field is a function of depth as the field of mass varies with depth. However the slope current which usually is a surface phenomenon, modifies the free water surface and produces only constant pressure changes below the region of piled-up water.

Where there is no pile-up of water pressure is a stratification phenomenon and the total pressure = the relative pressure as calculated from the slope of the isobars.

At a given depth measured from the water surface, the pressure beneath a pile up of water is less because of a deficiency of mass than at the same depth in a water column with no pile-up

(See illustration on next page)



$P_T(z) > P_t(z')$

$P_T(z) = P_T(z'')$

thus the isobars under a water pile-up are depressed at a distance dz vertically downward due to the mass deficiency. The total pressure at the given depth Z' is

$$P_T = P_{T(Z)} - \rho g dz$$

$$= P_R - \Delta P$$

in actual fact the water surface is not a geopotential surface because piled-up water being lighter floats above the field of varing mass. As the depression due to the floating piled-up water is a constant its effect is to shift the isobars in the field of varing mass downward without distortion. The depression dz can only be measured by precise leveling which is now only possible at great exprese and difficulty with large gyroscopic platforms. So the pressure gradients measured at sea only equal the total pressure gradients when the ocean above the measurement is in the varying field of mass.

In actual fact the total pressure gradient Relative pressure - correction press.

$$\frac{\partial P_T}{\partial N} = \left(\frac{\partial P_R}{\partial N} - g\rho \frac{dz}{dn} \right)$$

In the oceanographic co-ordinate system as Z is positive downward

$$-\frac{\partial P_T}{\partial N} = -\frac{\partial P_R}{\partial N} + g\rho \frac{dz}{dn}$$

where

$$\frac{dz}{dn} = \tan\theta = \text{slope of surface of piled up water.}$$

116

The Geostrophic Equations considering total pressure are:

$$2\omega \, \text{SIN} \, \phi \, Cy = \alpha \frac{\partial P_R}{\partial X} - g \tan \theta \, X$$
$$-2\omega \, \text{SIN} \, \phi \, Cx = \alpha \frac{\partial P_R}{\partial y} - g \tan \theta \, y$$

the total geostrophic velocity thus is the sum of velocities due to the relative pressure gradient and the slope gradient

$$C_T = C_R + C_S$$

or

$$f(C_{R,X} + C_{S,X}) = -\alpha \frac{\partial P_R}{\partial y} + g \tan \theta \, y$$
$$f(C_{R,y} + C_{S,y}) = \alpha \frac{\partial P_R}{\partial X} - g \tan \theta \, X$$

where

$$C_{R,X} = -\frac{\alpha}{f} \frac{\partial P_R}{\partial y}$$
$$C_{S,X} = \frac{g}{f} \tan \theta \, y$$
$$C_{R,y} = \frac{\alpha}{f} \frac{\partial P_R}{\partial X}$$
$$C_{S,y} = -\frac{g}{f} \tan \theta \, X$$

III. CALCULATIONS OF RELATIVE GEOSTROPHIC VELOCITY

As noted above only relative pressures can be calculated from knowledge of the field of mass, which can be determined from hydrographic stations.

$$dP = g\rho dz$$

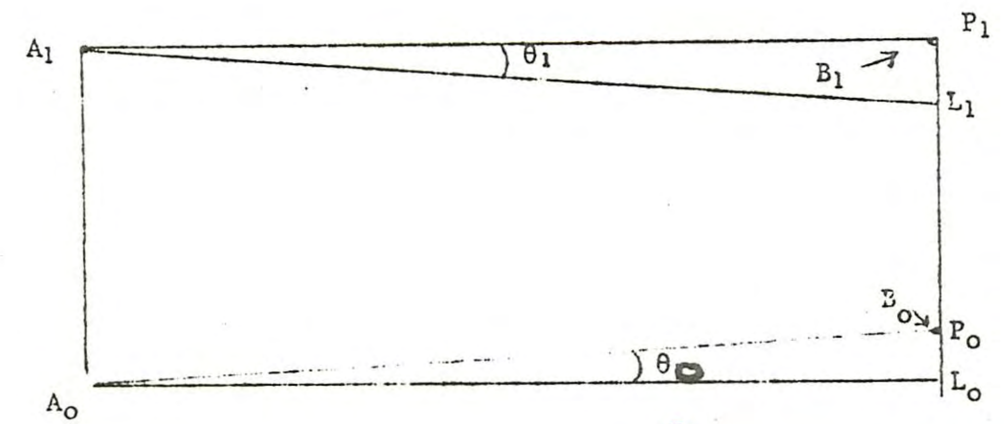
and

$$c = \frac{\alpha}{f} \frac{dP_R}{dn}$$
$$c = \frac{g\rho dz}{\rho f dn} = \frac{g}{f} \frac{dz}{dn}$$

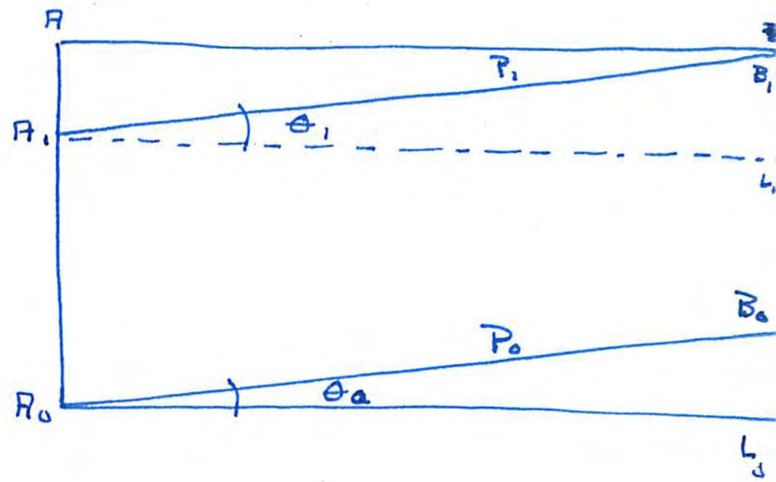
as $gdz \equiv dD$ $\qquad$ δ Dynamic Depth

$$c = \frac{dD}{fdn}$$

In practice it is impossible to know the direction of N at a station. Between the stations, which is taken as the distance N



5/b 2



$$\tan \theta_1 = \frac{B_1L_1}{A_1L_1} = \frac{B_1L_1}{AB}$$

$$\tan \theta_o = \frac{B_oL_o}{A_oL_o} = \frac{B_oL_o}{AB}$$

By By the gradient equation

$$C_1 = \frac{g}{f} \tan \theta_1$$

$$C_o = \frac{g}{f} \tan \theta_o$$

$$C_1 - C_o = \frac{g}{f} (\tan \theta_1 - \tan \theta_o)$$

$$= \frac{g}{f} \left(\frac{B_1L_1 - B_oL_o}{AB}\right)$$

$$\text{Dynamic Depth } D \equiv \int_A^B g\,dz$$

(Defining equation with g = gravity)

$$\text{AND } g(B_1L_1 - B_oL_o) = D_{L_1}^{B_1} - D_{L_o}^{B_o}$$

$$B_1L_1 - B_oL_o = (B_1L_1 + L_1B_o) - (B_oL_o + L_1B_o)$$

$$= B_oB_1 - L_oL_1$$

$$= B_oB_1 - A_oA_1$$

OR

$$C_1 - C_o = \frac{g}{f} \frac{(B_oB_1 - A_oA_1)}{AB}$$

$$= \frac{D_{B_1}^{B_o} - D_{A_1}^{A_o}}{f\,AB}$$

HELAND-HANSEN EQUATION

relative equation so look for place where C_o is 0 (called depth of no motion) and can then solve for C_1 exactly

AS

$$D_{B_1}^{B_o} = \int_{P_1}^{P_o} \alpha_B \, dP$$

$$D_{A_1}^{A_o} = \int_{P_1}^{P_o} \alpha_A \, dP$$

OR

$$C_1 - C_o = \frac{1}{2\omega \, \mathrm{SIN}\phi \, AB} \int_{P_1}^{P_o} (\alpha_B - \alpha_A) \, dP$$

difference in velocity for two isobars =

For ease in calculation the ocean is assumed to have a standard condition with a uniform salinity of 35 o/oo and a temperature of 0°C so that real values in the oceans are measured as differences from these standard conditions where

$$(D_1 - D_2)_{total}$$

$$(D_1 - D_2)\ \text{standard} + \Delta D =$$

$$\int_{P_1}^{P_2} \alpha_{35,0}\, p\,dP + \int_{P_1}^{P_2} \delta\, dP$$

the standard ocean is assumed to be motionless to that the geostrophic effects may be related to anomalous distributions of the specific volume δ

assume solution to H.H. equation is a relative solution

$$\Delta T = \Delta_{ST} = \delta_S + \delta_T + \delta_{ST}$$

409
SJF

$$(D_1 - D_2) = \int_{P_1}^{P_2} \alpha_{35,o,p} dP = \text{STANDARD GEOPOTENTIAL DISTANCE}$$ between isobaric surfaces P_1 and P_2

$$\Delta D = \int_{P_1}^{P_2} \alpha dP = \text{GEOPOTENTIAL ANOMALY}$$ ** anomaly of the geopotential surfaces between P_1 and P_2

$$\alpha_{S,T,P} - \alpha_{35,o,p} = \delta = \text{SPECIFIC VOLUME ANOMALY}$$

std. volume

δ is positive because the (standard ocean) is salty and cold enough to have a $\alpha_{35,0,P}$ value less than the in place specific volume $\alpha_{S,T,P}$.

$$\alpha_{35,0,P} = \alpha_{34,0,0} + \delta_p$$

$$\alpha_{35,0,0} = .97264 \text{ cm}^3/\text{gm}$$

δ_p considers the effect of pressure on the standard ocean.

$\alpha_{35,0,P}$ can be read directly from tables (see La Fond Table 4).

$$\delta = (\delta_S + \delta_T + \delta_{S,T}) + (\delta_{S,P} + \delta_{T,P} + \delta_{S,T,P})$$ (related to σ_T)

$\Delta_{S,T} = (\delta_S + \delta_T + \delta_{S,T})$ is the dependance of the specific volume anomaly on salinity and temperature (see La Fond Table 5a)

also called Δ_T = thermosteric anomaly

$$\alpha_{S,T,0} = \frac{1}{\rho_{S,T,o}} = \frac{1-10^{-3}\sigma T}{1+10^{-3}\sigma T}$$

$$\alpha_{S,T,0} = \alpha_{35,0,0} + \Delta_{S,T}$$

$$= 0.97264 + \Delta_{S,T}$$

$$\text{So } \Delta_{S,T} = 0.02736 - \frac{10^{-3}\sigma T}{1+10^{-3}\sigma T}$$

$\delta_{S,T,P}$ is so small it can be neglected.

Practically $\delta = \Delta_{S,T} + \delta_{s,p} + \delta_{T,P}$

All terms can be read from tables with a knowledge of depth, temperature, and salinity.

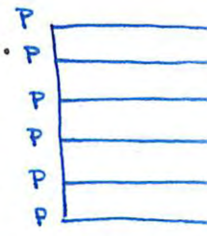
SEE SJF

UNIVERSITY OF CALIFORNIA
Fall Quarter 1970
CE 201 A

Department of Civil Engineering
Ocean Engineering
Instructor: Pat Wilde

THE DYNAMIC METHOD FOR VELOCITY
AND TRANSPORT VOLUME ESTIMATIONS

The above discussion on calculations of the specific volume anomaly was based on the concept of deviations of the field of mass from the standard ocean with salinity of 35 $^{o}/oo$ and a temperature of $0^{o}C$ at any given pressure. In effect the standard ocean consists of a vertical grid of stacked isobaric surfaces. As noted in discussion of the relationship among dynamic depth in dynamic meters, pressure in decibars, and depth in meters; their numerical values differ by only 2%. Thus the values of depth gives the pressure values to within 2%. Accordingly, in the standard ocean the pressure valves of the reference isobaric surfaces can be obtained from the depth and the standard ocean becomes an easily visualize grid. Now $\Delta D = \int_{P_1}^{P_2} \delta dP$ may be evaluated for appropriate intervals as pressure valves correspond to depth valves and the specific volume a nomaly δ can be computed from tables if temperature and salinity information is known in the interval.



In other words at a station the dynamic height is the intersection value of the relative field of mass and a reference grid isobaric surface, just as the geographic height is the intersection value of the ground surface with a reference level surface. A plot of dynamic height values for a series of stations could be contoured with reference to the same reference isobaric surface, continuing the geographic analogy. The "relief" with reference to the isobaric surface is called the DYNAMIC TOPOGRAPHY. The spacing of the isopleths of dynamic depth is related to the geostrophic velocity at the top of the chosen interval through the Helland-Hansen Equation.

work = mass $\times g \times h$

$$\frac{\text{work}}{\text{mass}} = g \times h = g\,\partial z = D = \text{gravity potential}$$
$$= \text{dynamic height}$$

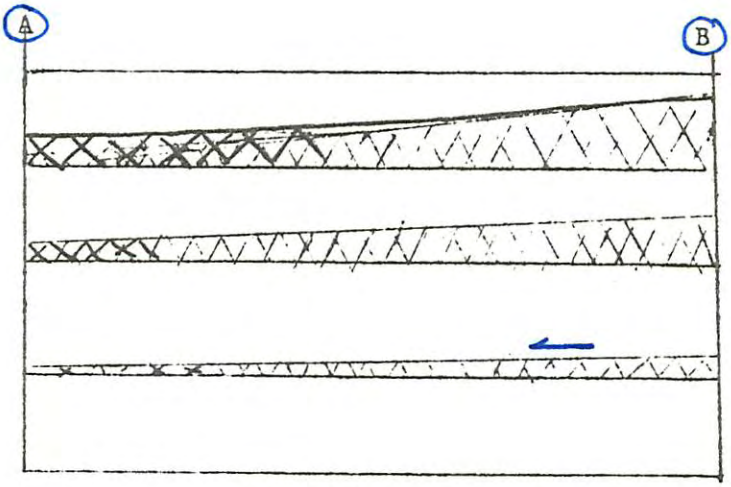
SEE page 129

(std. ocean has no inclined isobars)

$$\Delta D_A = \int_{P_2}^{P_N} \delta_A \, dP$$

$$\Delta D_A = \int_{P_3}^{P_N} \delta_A \, dP$$

$$\Delta D_A = \int_{P_4}^{P_N} \delta_A \, dP$$



$Z_1 = P_1$: $\alpha 35,0,P_1$ = constant

$Z_2 = P_2$ · $\alpha 35,0,P_2$ = constant

$Z_3 = P_3$; $\alpha 35,0,P_3$ = constant

$Z_4 = P_4$; $\alpha 35,0,P_4$ = constant

$Z_N = P_N$: $\alpha 35,0,P_N$ = constant

Relative velocities now may be computed from the Helland-Hansen equation:

STD. OCEAN

$$C_1 - C_N = \frac{1}{f\ AB} \left[\left(\int_{P_1}^{P_N} \alpha_B 35,\ 0,\ p dP + \int_{P_1}^{P_N} \delta_B dP \right) - \left(\int_{P_1}^{P_N} \alpha_A 35, 0,\ p dP + \int_{P_1}^{P_N} \delta_A dP \right) \right]$$

← specific vol. anomaly

$$\text{as} \int_{P_1}^{P_N} \alpha_B, 35,\ 0,\ p dP = \int_{P_1}^{P_N} \alpha_A\ 35,\ 0,\ p dP \text{ (Standard Ocean)}$$

$$\text{then } C_1 - C_N = \frac{1}{f\ AB} \int_{P_1}^{P_N} \delta_B dP - \int_{P_1}^{P_N} \delta_A \, dP = \frac{1}{f\ AB} \Delta D_B - \Delta D_A$$

Appendix D-1 illustrates a sample calculation of Dynamic depths and relative velocities from data derived from hydrographic stations. Most available hydrographic station information comes from Nansen bottle casts so there is no continuous profile of temperature and salinity and thus no continuous profile of the specific volume anomaly. Thus in practice mean values of δ and the derived ΔD must be computed between data points. This leads to uncertainties

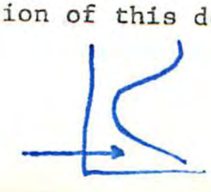
relative method so can be no pile up of water on surface such as at mouth of rivers in the velocity calculations especially if there are large vertical intervals between nansen bottles. The specific volume anomaly becomes smaller at depth as the density of the real ocean approaches that of the standard ocean. Therefore the spacing of the nansen bottles is usually closer near the surface where deviations from the standard ocean are largest. This procedure reduces the uncertainty in averaging δ values between bottles. Eventually this problem will be solved as more stations are made with the continuously profiling T-D-S device.

DEPTH OF NO MOTION PROBLEM

Once a compilation of dynamic depths is made the next question is the choice of the reference P_N so the dynamic topography can be constructed. Obviously it would be advisable to choice P_N such that $V_N = 0$ so the relative velocity calculated from the Helland-Hansen equation would be actually an absolute velocity. As in practice the value for depth is substituted to the value of pressure the search for an appropriate reference datum is called "The Depth of no Motion Problem" refering to the depth at which there is no horizontal geostrophic motion. Ideally the depth of no motion could be determined directly by current meter measurements or from detailed tracing of neutrally buoyant buoys. However, if one had the vertical profile of velocity directly one wouldn't need to make dynamic calculations!!

Below are listed some methods by which the depth of no motion is determined indirectly.

1. Correlating the depth of no motion with the oxygen minimum (Dietrich, 1936) this method assumes that biological activity is independant of depth so that minimum oxygen content implies minimal replentishment of low horizontal movement. The Oxygen minimum occurs at reasonably shallow depths, usually less than 1000 meters, so selection of this depth as a reference produces counter



oxygen minim occurs when dead organisms are oxidized - after mini. deep water has O_2 which moves in from the areas currents. Use of this method means that oxygen valvue must be taken along with temperature and salinity data but this is usually done anyway as a standard practice.

2. Correlating the depth of no motion with the depth of minimum distortion of isopycnal layers. Parr (1938) noted that for two stations with a significant current between them the isopycnal surfaces must be displaced vertically just as isobaric surfaces would be displaced. The depth where the isopycnals are least displaced should correspond to the region of maximum stability or the depth of minimal motion.

3. Correlating the depth of no motion with depth of minimal horizontal salt diffusion (Hidaka, 1949). The salinity distribution in the oceans is related to:

$$\frac{ds}{dt} = K_1\frac{\partial^2 S}{\partial X^2} + K_2\frac{\partial^2 S}{\partial Y^2} + K_3\frac{\partial^2 S}{\partial Z^2}$$

Presumbly at the depth of no motion: $\frac{ds}{dt} = 0$

So for dominant horizontal diffusion: $\frac{\partial^2 S}{\partial X^2} + \frac{\partial^2 S}{\partial Y^2} = 0$

And for dominant vertical diffusion: $\frac{\partial^2 S}{\partial Z^2} = 0$

Assuming the diffusion co-efficients are equal. Hidaka used the depth where $\frac{\partial^2 S}{\partial Z^2} \longrightarrow 0$ as the depth of minimal notion reasoning that maximum horizontal diffusion motion. implies minimal horizontal diffusion & little horizontal

3. Computation of the absolute vertical distribution by dynamic calculations and the continunity equations (Hidaka, 1950). This method utilizes a minimum of four hydrographic stations and computes the lateral transport of water and salt through the six vertical interfaces between the stations. As the Helland-Hansen equation only gives relative velocities

$C_{A(Z)} - C_N = C_{R(Z)}$ where

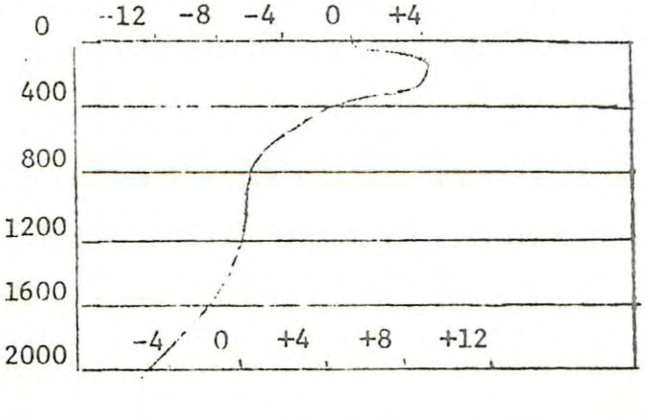
$C_{A(Z)}$ = absolute geostrophic velocity at some depth Z

$C_{R(Z)}$ = Relative geostrophic velocity compared to some reference datum

C_N = Absolute geostrophic velocity at some reference datum

Rearranging $C_A = C_R + C_N$ C_R is computed from hydrographic data and C_N is found by solving the six interface continunity equations for the surface velocity.

(5) Correlating the depth of no motion with the zone of near constant dynamic depth differences (Defant, 1941) a plot of pressure on the ordinate versus dynamic depth differences $(\Delta D_B - \Delta D_A)$ on the upper abcissa generally show an upper region of changing $(\Delta D_B - \Delta D_A)$ overlying a zone of relatively constant $(\Delta D_B - \Delta D_A)$, which in turn in many areas overlies a lower region of varying $(\Delta D_B - \Delta D_A)$. This plot gives the relative vertical distribution of velocity when the appropiate f valve for the latitude of the stations and the distance between the stations (both constant for fixed stations) are substituted into the Helland-Hansen equation. Defant argued that the logical placing of a zero displacement is in line with the zone of constant $(\Delta D_B - \Delta D_A)$ to give a reasonable distribution of absolute velocities.



Relative Dynamic Depths $(\Delta D_B - \Delta D_A)$

(After Defant, 1941)

Absolute Dynamic Depths

None of these five methods are free of uncertainties as shown in Formin's (1964, p. 117-148) excellent detailed discussion of the depth of no motion problem. Defant's method has the fewest theoretical objections to it and is the most commonly used method to select the reference surface for dynamic calculations.

TRANSPORT CALCULATIONS

The quantity of water passing between two hydrographic stations ideally is:

$$Q = AB \int_{Z_2}^{Z_3} \int_{C_3}^{C_2} dc\,dz$$

WHERE

AB = distance between stations

However, as C is some complex function of α and Z practically it is advisable to use the summation of small intervals where some mean velocity would be a valid approximation or

$$Q = AB \int_{Z_2}^{Z_3} \bar{c}\; d\,z$$

if we make the further assumption that the depth of no motion is a geopotential surface or $C_N = 0$ at some Z_N the Helland-Hansen equation reduces to:

$$C = \frac{1}{fAB}\left[\Delta D_B - \Delta D_A\right] \quad \underline{\text{or}}\ Q = \int_{Z_2}^{Z_N} \frac{1}{f}\left[\Delta D_B - \Delta D_A\right]dz$$

Thus it is possible to calculate the transport between two stations without knowing the distance between them as long as the latitude is known and a dynamic section between the two stations can be constructed. Again it is sensible to keep the intervals small especially if there are reversals in the slope of dynamic topography (or isosteres of specific volume) because if reversals of current direction occur within the interval, the above formula will only give the total transport between stations, which is misleading in terms of oceanic circulation.

If some non-geopotential surface is chosen as the surface of no-motion (for example the oxygen minimum) the above simplified rectangular unit equation is not valid. Graphic solutions, thus, must be used at the shapes of the units becomes comnlex or $Z_{N(A)} \neq Z_{N(B)}$.

In most of the non level assumptions of the depth of no motion, the reference surface is taken shallow enough so that there is a reversal of slope of the dynamic topography below the reference surface. Thus small intervals and careful observance of the sign of $(\Delta D_B - \Delta D_A)$ is necessary to prevent gross errors in transport calculations and prevent calculation of erroneous direction of flow.

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References

Defant, A., 1941, Die Absolute Topographie des Physikalischen Meeresniveaus uno der Druckflachen, Sowie, Die Wasser Bewegungen im Atlantischen Ozean: Deut, Atl. Exp. "Meteor", 1925-1927, V. 6, Part 2, p. 191-260.

Dietrich, G., 1936, Aufbau and Bewegung von Golfstrom und Agulhasstrom: Naturwissenshaften No. 15.

Fomin, L. M., 1964, The Dynamic Method in Oceanography: Amsterdam, Elsevier, 212 p.

Hidaka, K. 1949, Depth of Motionless Layer as Inferred from Distribution of Salinity in the Oceans: Trans. Am. Geophys. Union, V. 30.

Hidaka, K., 1950, An Attempt to Determine the Motionless Layers in the Ocean: Oceanographic Mag, No. 2.

Park, A., 1938, Analysis of Current Profiles by a Study of Pycnometric Distortion and Identifying Properties: Jour. Marine Res., V. 1.

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UNITS FOR DYNAMIC CALCULATIONS

Unquestionably the most difficult part of dynamic calculations of velocity and transport is the correct substitution of units in the Helland-Hansen equation. This basically stems from the fact that the fundamental unit of gravity potential in the MKS system is the dynamic decimeter whereas the practical unit is the dynamic meter which is 10 times larger but has the same units M^2/SEC^2. The following arguement will attempt to demonstrate the fundamental assumptions underlying practical use of the gravity potential.

WORK - GRAVITY POTENTIAL AND DYNAMIC UNITS

$\frac{work}{mass} = gh = D$

$\delta D = g \delta z$

As work = mass X gravity X height
work per unit mass $w/m = gh$ which is defined as the gravity potential. In the MKS system the unit mass is the kilogram (Kg), height or for oceanographic purposes depth (z) is in meters (M) and the acceleration due to gravity g is approximately 10 M/sec. $g = 9.8\ M/SEC^2$
Thus the gravity potential to move vertically one Kg one meter =

$$GP = (10\ M/SEC^2)M = 10\ M^2/SEC^2 = D_M \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

Where D_M is called the $\boxed{\text{dynamic meter.}}$ However, to obtain a unit value of the gravity potential one must move the mass a less distance as the value of g is reasonably fixed or

$$GP = (10M/SEC^2)\ .1M = M^2/SEC^2 = D_D \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

As .1 is 1 decimeter, D_D is called the $\boxed{\text{dynamic decimeter}}$ whose value is 1/10 of D_M Obviously g is not exactly 10 M/SEC^2 but closer to 9.8 M/SEC^2. Thus the actual geometric distance the unit mass is moved is somewhat greater than 1 decimeter so the names of the dynamic units are somewhat misleading. However, as g is a constant at any given location over a small change in the vertical the relationship between these dynamic units is still $D_M = 10D_D$.

Pressure

$$\partial P = \rho g \partial z = \rho \partial D$$

$$\partial D = \alpha \partial P$$

Depth (Z) in meters =N

Pressure (P) in decibars =N

Z = 10 meters

P = 1 Bar = 10 decibars

D_m = 10 dynamic meters

D_D = 1 dynamic deci

numerically equal, units are not

numbers that represent the geometric depth in meters = absolute pressure in decibars

The dynamic units also may be written in another form.

$$\text{As } 1 \text{ Newton} = \text{Kg} \cdot \text{M/SEC}^2 \quad \ldots \quad (3)$$

$$D = 10\ \text{M}^2/\text{SEC}^2 = \frac{10 \text{ Newton} \cdot \text{M}}{\text{Kg}} \quad \ldots \quad (4)$$

or

$$\text{Newton} = \frac{D_M \cdot \text{Kg}}{10\ \text{M}} = \frac{D_M \cdot 10^3 \text{gm}}{10^3 \text{cm}} = \frac{D_M\ \text{gm}}{\text{cm}} \quad \ldots \quad (5)$$

and

$$D_D = \text{M}^2/\text{SEC}^2 = \frac{\text{Newton} \cdot \text{M}}{\text{Kg}} \quad \ldots \quad (6)$$

or

$$\text{Newton} = \frac{D_D \cdot \text{Kg}}{\text{M}} = \frac{D_D \cdot 10^3 \text{gm}}{10^2 \text{cm}} = \frac{10 D_D\ \text{gm}}{\text{cm}} \quad \ldots \quad (7)$$

RELATIONSHIP BETWEEN DYNAMIC UNITS AND PRESSURE

In oceanographic studies the useful unit of pressure is the atmosphere which is approximated by

$$1 \text{ ATM} \simeq 1 \text{ BAR} = 10^6 \text{ Dynes/cm}^2 \quad \ldots \quad (8)$$

as

$$1 \text{ Newton} = \text{kg} \cdot \text{M/SEC}^2 = 10^5 \text{ gm} \cdot \text{cm/SEC}^2 = 10^5 \text{ Dynes} \quad \ldots \quad (9)$$

$$1 \text{ BAR} = 10 \text{ Newtons/cm}^2 \quad \ldots \quad (10)$$

or

$$1 \text{ Decibar} = \text{Newton/cm}^2 \quad \ldots \quad (11)$$

Substitution of (5) into (11) yields:

$$1 \text{ Decibar} = \frac{D_M\ \text{gm}}{\text{cm}^3} = D_M \rho \quad \ldots \quad (12$$

In oceanographic cases the density (ρ) of sea water is approximately 1 so numerically the value of the pressure in Decibars equals the value of the gravity potential in dynamic meters.

More precisely EQ (12) may be written with ρ as a constant:

$$Dm = \int_P^P \frac{1}{\rho} dP = \int_P^P \alpha dP \quad \ldots\ldots (13)$$

for P in Decibars and α in cm^3/Gram

in like manner

$$D_D = \int_P^P \alpha dP \quad \ldots\ldots (14)$$

But ρ must be in centibars (.01 BARS). As the depth in meters is essentially equal numerically to the value of the pressure in decibars the practical oceanographic relationship between pressure and gravity potential is given by EQ (13) with units Decibars, dynamic meters, cm^3/gram. But one must remember the units of dynamic meters by EQ (1) are 10 M^2/SEC^2 NOT M^2/SEC^2 so conversion to cm^2/SEC^2 is

$$D_m = 10\ m^2/SEC^2 = \frac{10^5 cm^2}{SEC^2} \quad \ldots\ldots (15)$$

SUBSTITUTION IN THE HELLAND-HANSEN EQUATION

The Helland-Hansen equation may be written in various forms

$$C_1 - C_0 = \frac{1}{f\ AB} (\Delta D_B - \Delta D_A) \quad \ldots\ldots (16)$$

$$= \frac{1}{f\ AB} \left[\int_{P_1}^{P_0} \delta_B\, dP - \int_{P_1}^{P_0} \delta_A df \right] \quad \ldots\ldots (17)$$

$$\text{numerically} = \frac{1}{f\ AB} \left[\int_{z_1}^{z_0} \delta_B dz - \int_{z_1}^{z_0} \delta_E dz \right] \quad \ldots\ldots (18)$$

Usually EQ (18) is used with the units $\frac{1}{f\ AB}$ in $\frac{SEC}{cm}$

z in meters

δ in cm^3/gm

to yield $C_1 - C_0$ in cm/SEC. However tables of the various δ anomalies for example Sverdrup and others (1942, p. 1051-1059) and Bialek (1966, p. 102-318) are given in $10^5\Delta$ So multiplication by the internal depth in meters to solve EQ (16) from EQ (18) gives ΔD directly in cm^2/SEC^2. To obtain ΔD in dynamic meters one must divide the result of integral part (18) by 10^5 to give ΔD in D_m or $10m^2/SEC^2$.

Direct evaluation of the specific volume anomaly is possible by solution of the equation developed by Wilson and Bradley (1968 p. 361) for both δSTP and for $\delta 35, 0, P$ or

$$\delta = \delta STP - \delta 35,0,P \quad (19)$$

$$\delta = 0.70200 + \left[\frac{100(17.5273 + 0.1101t - 0.000639t^2 - 0.039986SAL - 0.000107\ (SAL)^2)}{(P + 5880.9 + 37.592t - 0.34395t + 2.2524SAL)}\right]$$

$$- \left[\ 0.70200 + \frac{(1612.779)}{(P + 5880.9 + 78.8340)}\right] \quad (20)$$

For salinity (SAL) in 0/00 temperature (t) in degrees centigrade pressure(P) in Bars, P numerically equal to depth in Dekameters or depth in meters /10. *
δ in cm^3/gm.
Solving equation (18) by substitution of z in meters and EQ (20) will yield ΔD of EQ. (16) in dynamic meters; thus the units of $C_1 - C_0$ will be 10 M/SEC, for (AB) also in meters.

* Actually P = (meters)/9.8

REFERENCES

Bialek, E.L., 1966, Handbook of Oceanographic Tables: U.S. Naval Ocean. Spec. Pub. SP-68, 427p.

Bjerknes, U., and Sandstrom, J.W., 1910, Dynamic Meteorology and Hydrography; Carnegie Inst. Wash. Pub. 88, Part I Statics, 23 p..

Sverdrup, H.U., Johnson, M.W., and Fleming, R.H., 1942, The Oceans: Englewood Cliffs, New Jersey, Prentice-Hall, 1087 p.

Wilson, W. and Bradley, D., 1968, Specific Volume of Sea Water As a Function of Tenperature, Pressure and Salinity: Deep-Sea Res., V.15, p. 355-363.

CE 201A
Ocean Engineering
Physical Oceanology
Appendix D 1

Department of Civil Engineering
Division of Hydraulic & Sanitary Engr.
Instructor: P. Wilde

SAMPLE CALCULATION OF DYNAMIC PROPERTIES

Station A	54°02'N	24°34'W
Station B	54°06'N	23°01'W

Distance AB = 1.03×10^5 meters

$f = 1.18 \times 10^{-4}$/sec

$$C_1 - C_n = \frac{1}{f\overline{AB}} \left(\int_{P_1}^{P_n} \delta_B \, dP - \int_{P_1}^{P_n} \delta_A \, dP \right)$$

$$Q = AB \int_{Z_2}^{Z_3} \bar{c} \, dz$$

TABLE D-1 STATION A

from tables after knowing σT

$\Delta D = \int_{P_1}^{P_2} \delta \, dP$

Depth Meter	Temp. °C	Salinity 0/00	σ_t	$10^5\Delta_{st}$	$10^5\delta_{sp}$	$10^5\delta_{tp}$	$10^5\delta$ SUM	Mean $10^5\delta$	ΔD Dynamic Meters	Σ ΔD
0	12.70	35.02	26.49	155	0	0	155			
								141.5	.0354	
25	11.28	.06	.79	127	0	1	128			.0354
								111.5	.0288	
50	9.68	.14	27.14	94	0	1	95			.0642
								86.0	.0430	
100	8.13	.09	.34	75	0	2	77			.1072
								73.0	.0735	
200	7.53	.08	.42	67	0	3	70			.1807
								66.5	.1330	
400	5.73	34.90	.53	57	0	6	63			.3137
								57.5	.1150	
600	5.02	.95	.65	45	0	7	52			.4287
								49.5	.0990	
800	4.04	.90	.72	39	0	8	47			.5277
								47.5	.0950	
1000	3.97	.89	.73	38	0	10	48			.6227
								45.5	.0910	
1200	3.54	.89	.78	33	0	10	43			.7137
								44.0	.0880	
1400	3.40	.89	.78	33	0	12	45			.8017
								45.5	.0910	
1600	3.29	.88	.78	33	0	13	46			.8927
								46.5	.0930	
1800	3.35	.89	.79	32	0	15	47			.9857
								47.5	.0950	
2000	3.41	.90	.79	32	0	16	48			1.0807

$10^5 \times \Delta_{st} = 155$, etc.

TABLE D-2 STATION B

Depth Meter	Temp. °C	Salinity 0/00	σ_t	$10^5\Delta_{st}$	$10^5\delta_{sp}$	$10^5\delta_{tp}$	$10^5\delta$	Mean $10^5\delta$	ΔD Dynamic Meters	$\sum$ ΔD
0	12.70	35.30	26.70	135	0	0	135			
25	12.25	.34	.82	124	0	1	125	130	.0325	.0325
50	10.20	.35	27.13	95	0	1	96	110.5	.0263	.0588
100	9.59	.34	.31	78	0	2	80	88.0	.0440	.1028
200	9.16	.32	.36	73	0	4	77	78.5	.0785	.1813
400	8.96	.26	.35	74	0	8	82	79.5	.1590	.3403
600	8.22	.17	.39	70	0	11	81	81.5	.1630	.5033
800	6.45	.03	.54	56	0	12	68	74.5	.1490	.6523
1000	5.08	34.99	.68	43	0	12	55	61.5	.1230	.7753
1200	4.19	.96	.76	35	0	12	47	51.0	.1020	.8773
1400	3.83	.93	.76	35	0	13	48	47.5	.0950	.9723
1600	3.71	.90	.76	35	0	14	49	48.5	.0970	1.0693
1800	3.50	.89	.77	34	0	15	49	49.0	.0980	1.1673
2000	3.31	.88	.78	33	0	16	49	49.0	.0980	1.2653

TABLE D - 3

Depth Meters	$\Delta D_B - \Delta D_A$ cm^2/sec^2	$\Delta D_B - \Delta D_A$ 2000db Datum	$V_1 - V_{2000}$ cm/sec	$\bar{V}$ cm/sec	Q $10^6 m^3/sec$	Σ Q* $10^6 m^3/sec$
0		18,460	15.2		0	0
	- 290 slope other way			15.3		
25		18,750	15.4		.39	- .39
	- 250			15.5		
50		19,000	15.6		.40	- .79
	+ 100			15.55		
100		18,900	15.5		.79	+ 2.37
	+ 500			15.3		
200		18 400	15 1		1.58	+ 5.26
	+2600			14.05		
400		15,800	13.0		2.89	+ 7.64
	+4800			11.55		
600		11,000	9.1		2.38	+ 9.09
	+5000			7.0		
800		6,000	4.9		1.45	+ 9.87
	+2800			3.75		
1000		3,200	2.6		.78	+10.34
	+1100			2.25		
1200		2,100	1.7		.47	+10.63
	+ 700			1.4		
1400		1 400	1.1		.29	+10.82
	+ 600			0.9		
1600		800	0.7		.19	+10.91
	+ 500			0.45		
1800		300	0.2		.09	+10.93
	+ 300			0.1		
2000			0.0		.02	

*(-) value flow to south; (+) value flow to north

University of California, Berkeley
College of Engineering
Department of Civil Engineering

OCEAN ENGINEERING

CE 201A PHYSICAL OCEANOLOGY

Dynamic Method Problem

GIVEN: Data Sheets for Two Stations

Spheres I
Mai-Tai 10

Klein Graphs

ASSUME: Mai-Tai Station 30 nautical miles west of Spheres Station on same parallel of latitude

DO:

a. Plot data on Klein Graph for each station.

b. From graph, complete data sheets.

c. Calculate velocity and transport profiles for depth of no motion

1. at bottom
2. at 2000 meters
3. at oxygen minimum

Present 1, 2, and 3 data in tables and graphically. Note and justify all assumptions.

(2)

OCEANOGRAPHIC DATA SUMMARY

UNIVERSITY OF CALIFORNIA
SCRIPPS INSTITUTION OF OCEANOGRAPHY

CRUISE: SPHERES

STATION: 10

VESSEL	STATION	MONTH	DAY	YEAR	HOUR (GCT)	LATITUDE °N	LONGITUDE °W	SOUNDING (F)	WIND DIR / FORCE	BAROMETER	TEMPERATURE °F DRY BULB / WET BULB	WEATHER	CLOUDS TYPE / AMT.	SEA	SWELL DIR / HT. / PERIOD	WIRE ∠
HORIZON	10	V	1	1966	1935 1855 0020	18° 49'	168° 31'			30.05		02				00 08 08

OBSERVED

Z (DEPTH) (METERS)	T (°C)	S (‰)	O_2 (ml/L)				δ_T (10^{-5} cm³/g)
0	25.5	34.844	4.63				
10	25.5	34.861	4.31				
30	25.6	34.846	4.75				
59	25.7	34.850	4.61				
79	25.5	34.905	4.45				
99	25.2	35.045	4.61				
123	24.0	35.210	4.66				
148	22.9	35.121	4.58				
198	20.3	34.971	4.27				
247	16.1	34.593	4.25				
247	16.14	34.683	4.31				
346	10.71	34.214	3.81				
499	6.77	34.202	1.53				
691	5.30	34.384	0.95				
936	4.35	34.510	1.22				
1178	3.57	34.552	1.28				
1470	2.88	34.583	1.64				
1764	2.39	34.612	1.96				
2060	2.08	34.635	2.15				
2354	1.85	34.649	2.51				
2408	1.77	34.654	2.55				
2702	1.[illegible]8	34.655	2.78				

INTERPOLATED / COMPUTED

Z (DEPTH) (METERS)	T (°C)	S (‰)	O_2 (ml/L)				σ_t (mg/cm³)	δ_T (10^{-5} cm³/g)	ΔD (DYN M)	
0										
10										
20										
30										
50										
75										
100										
125										
150										
200										
250										
300										
400										
500										
600										
700										
800										
1000										
1200										
1500										
2000										
2500										
3000										
4000										

OCEANOGRAPHIC DATA SUMMARY

UNIVERSITY OF CALIFORNIA
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CRUISE

STATION

VESSEL	STATION	MONTH	DAY	YEAR	HOUR (GCT)	LATITUDE °N	LONGITUDE °W	SOUNDING (F)	WIND DIR. FORCE	BAROMETER	TEMPERATURE °F DRY BULB WET BULB	WEATHER	CLOUDS TYPE AMT.	SEA	SWELL DIR. HT. PERIOD	WIRE ∠
HORIZON	10	V	1	1966												

OBSERVED

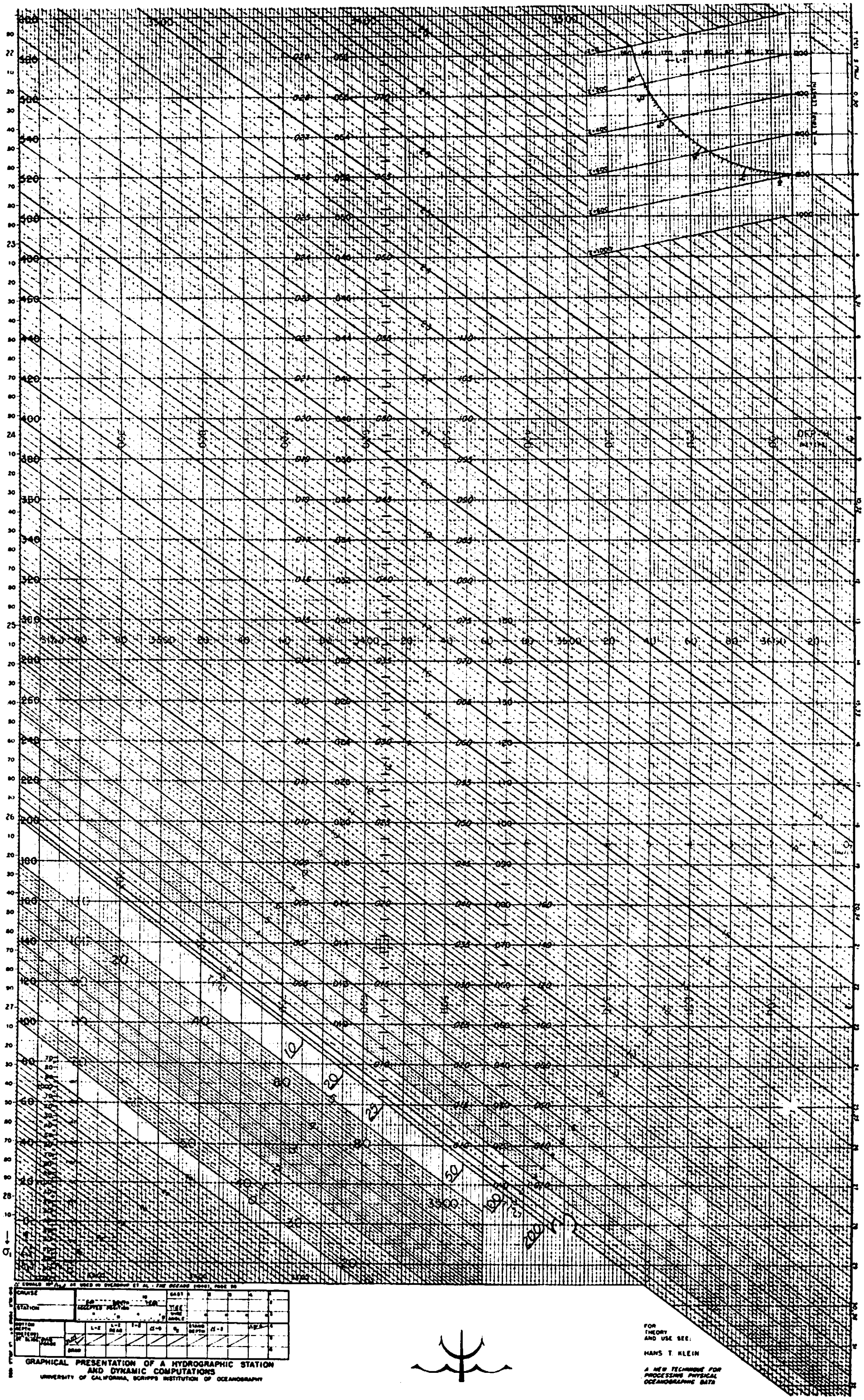
Z (DEPTH) (METERS)	T (°C)	S (‰)	O_2 (ml/L)				δ_T ($10^{-5}cm^3/g$)
2995	1.58	34.671	2.93				
3287	1.52	34.676	3.25				
3484	1.50	34.666	3.39				
3879	1.47	34.637	3.32				
4173	1.46	34.678	3.92				
4469	1.44	34.642	3.80				
4764	1.47	34.683	3.87				

INTERPOLATED / COMPUTED

Z (DEPTH) (METERS)	T (°C)	S (‰)	O_2 (ml/L)				σ_t (mg/cm³)	δ_T ($10^{-5}cm^3/g$)	ΔD (DYN M)
0									
10									
20									
30									
50									
75									
100									
125									
150									
200									
250									
300									
400									
500									
600									
700									
800									
1000									
1200									
1500									
2000									
2500									
3000									
4000									

SIO-MLR, FORM

LA JOLLA, 1953


GRAPHICAL PRESENTATION OF A HYDROGRAPHIC STATION
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OCEANOGRAPHIC DATA SUMMARY

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CRUISE: MAI-TAI 6609-A1

STATION: 1

VESSEL	STATION	MONTH	DAY	YEAR	HOUR (GCT)	LATITUDE °N	LONGITUDE °W	SOUNDING (F)	WIND DIR	WIND FORCE	BAROMETER	TEMPERATURE °F DRY BULB	TEMPERATURE °F WET BULB	WEATHER	CLOUDS TYPE	CLOUDS AMT.	SEA	SWELL	W.R.G.
HORIZON	1	IX	06	19 66	0037 0129 0349	18° 49.5	168° 31.0	2668	060°	3	29 97	82 2	78 5	02 overcast	6	8 8		missing	02 03 03

OBSERVED

Z (meters)	T (°C)	S (‰)	O_2 (ml/L)				δ_T ($10^{-5}cm^3/g$)
0	27.82	34.943	4.58				
30	27.76	34.933	4.63				
49	27.72	34.947	4.75				
74	26.21	34.945	4.95				
98		35.029	4.89				
196	18.65	34.893	4.28				
294	12.62	34.342	4.33				
391	8.73	34.196	2.45				
490	7.02	34.254	1.27				
686	5.59	34.414	0.86				
884	4.77	34.489	1.05				
1081	4.12	34.530	1.33				
1278	3.55	34.559	1.50				
1472	3.01	34.579	1.83				
1669	2.57	34.605	1.92				
1865	2.28	34.621	2.17				
2065a	2.13	34.631	2.33				
2261	1.98	34.643					
2457	1.84	34.653	2.59				
2653	1.76	34.659	2.71				
2138b	2.08	34.635	2.37				
2433	[illegible]	34.652	2.52				

INTERPOLATED / COMPUTED

Z (meters)	T (°C)	S (‰)	O_2 (ml/L)				σ_t (mg/cm^3)	δ_T ($10^{-5}cm^3/g$)	ΔD (dyn m)	
0										
10										
20										
30										
50										
75										
100										
125										
150										
200										
250										
300										
400										
500										
600										
700										
800										
1000										
1200										
1500										
2000										
2500										
3000										
4000										

OCEANOGRAPHIC DATA SUMMARY

UNIVERSITY OF CALIFORNIA
SCRIPPS INSTITUTION OF OCEANOGRAPHY

CRUISE ~~6609-A1~~
STATION 1

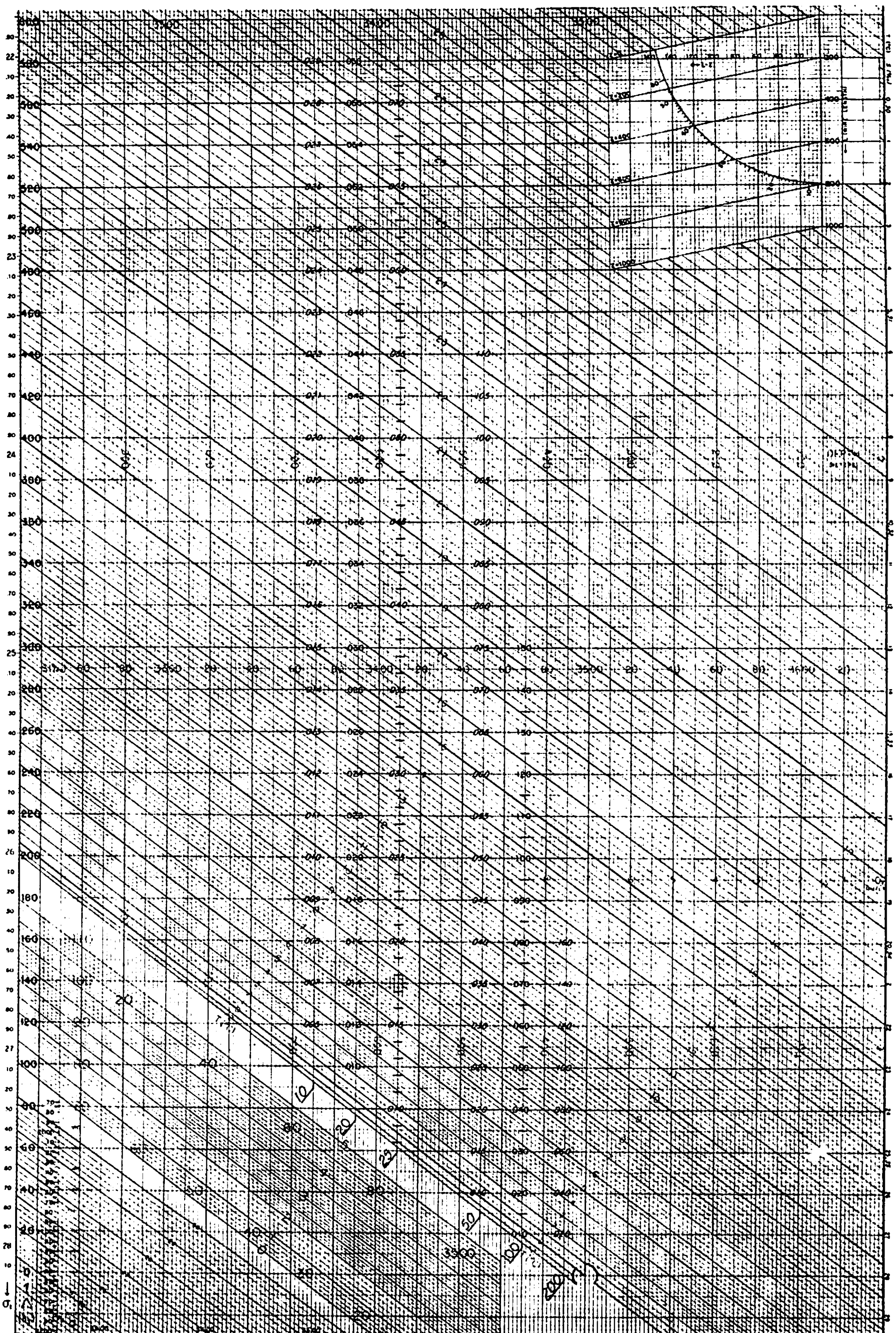
VESSEL: HORIZON | STATION: 1 | MONTH | DAY | YEAR: 19 | HOUR (GCT) | LATITUDE °N | LONGITUDE °W | SOUNDING (F) | WIND DIR FORCE | BAROMETER | TEMPERATURE °F DRY BULB WET BULB | WEATHER CLOUDS TYPE AMT. | SEA | SWELL DIR. HT. FT. PERIOD S. | WIRE ∠

OBSERVED

Z (DEPTH) (METERS)	T (°C)	S (‰)	O_2 (ml/L)				δ_T ($10^{-5}cm^3/g$)
2728	1.71	34.662	2.85				
3022	1.64	34.670	2.96				
3218	1.58	34.676	3.07				
3414	1.54	34.679	3.16				
3559	1.53	34.686	3.25				
3707	1.50	34.683	3.35				
3805	1.49	34.685	3.44				
3904	1.49	34.685	3.40				
4002	1.51	34.688	3.44				
4099	1.49	34.689	3.47				
4197	1.50	34.688	3.52				
4295	1.49	34.689	3.53				
4394	1.50	34.689	3.61				
4492	1.49	34.694	3.64				
4589	1.49	34.692	3.72				
4787	1.46	34.694	3.91				
4973	1.39	34.703	4.17				
4983	1.38	34.701	4.11				

INTERPOLATED / COMPUTED

Z (DEPTH) (METERS)	T (°C)	S (‰)	O_2 (ml/L)				σ_t (mg/cm³)	δ_T ($10^{-5}cm^3/g$)	ΔD (DYN. M)
0						0			
10						10			
20						20			
30						30			
50						50			
75						75			
100						100			
125						125			
150						150			
200						200			
250						250			
300						300			
400						400			
500						500			
600						600			
700						700			
800						800			
1000						1000			
1200						1200			
1500						1500			
2000						2000			
2500						2500			
3000						3000			
4000						4000			



GRAPHICAL PRESENTATION OF A HYDROGRAPHIC STATION AND DYNAMIC COMPUTATIONS

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113

UNIVERSITY OF CALIFORNIA
CE 201A

DEPARTMENT OF CIVIL ENGINEERING
Division of Hydraulic & Sanitary Engr.
Instructor: P. Wilde

DYNAMIC METHOD PROBLEM

Given: Data Sheets for Stations

CCOFI	6407 - 60.190
CCOFI	6407 - 63.200

Do:

a. From graphs or tables, complete data sheets for σ_T, δ_T, ΔD, use interpolated z.

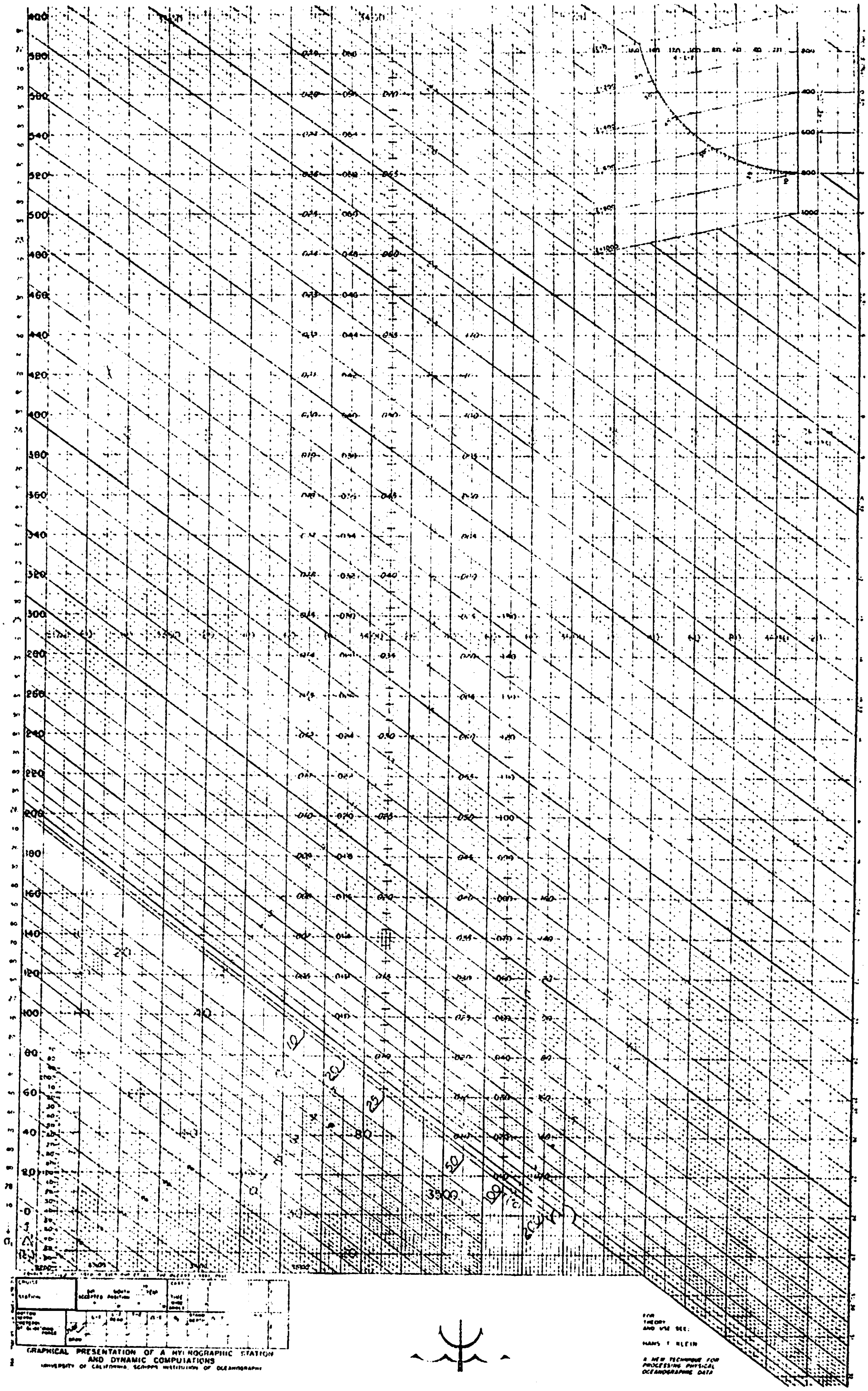
b. Calculate velocity and transport profiles for depth of no motion.

1. at bottom,
2. at 2,000 meters,
3. at oxygen minimum, plot each graphically.

c. Plot PHO (phosphate μg/L) and SIL (silicon μg/L) versus depth; discuss.

Show all work clearly. Note and justify all assumptions. Reference source of tables.

111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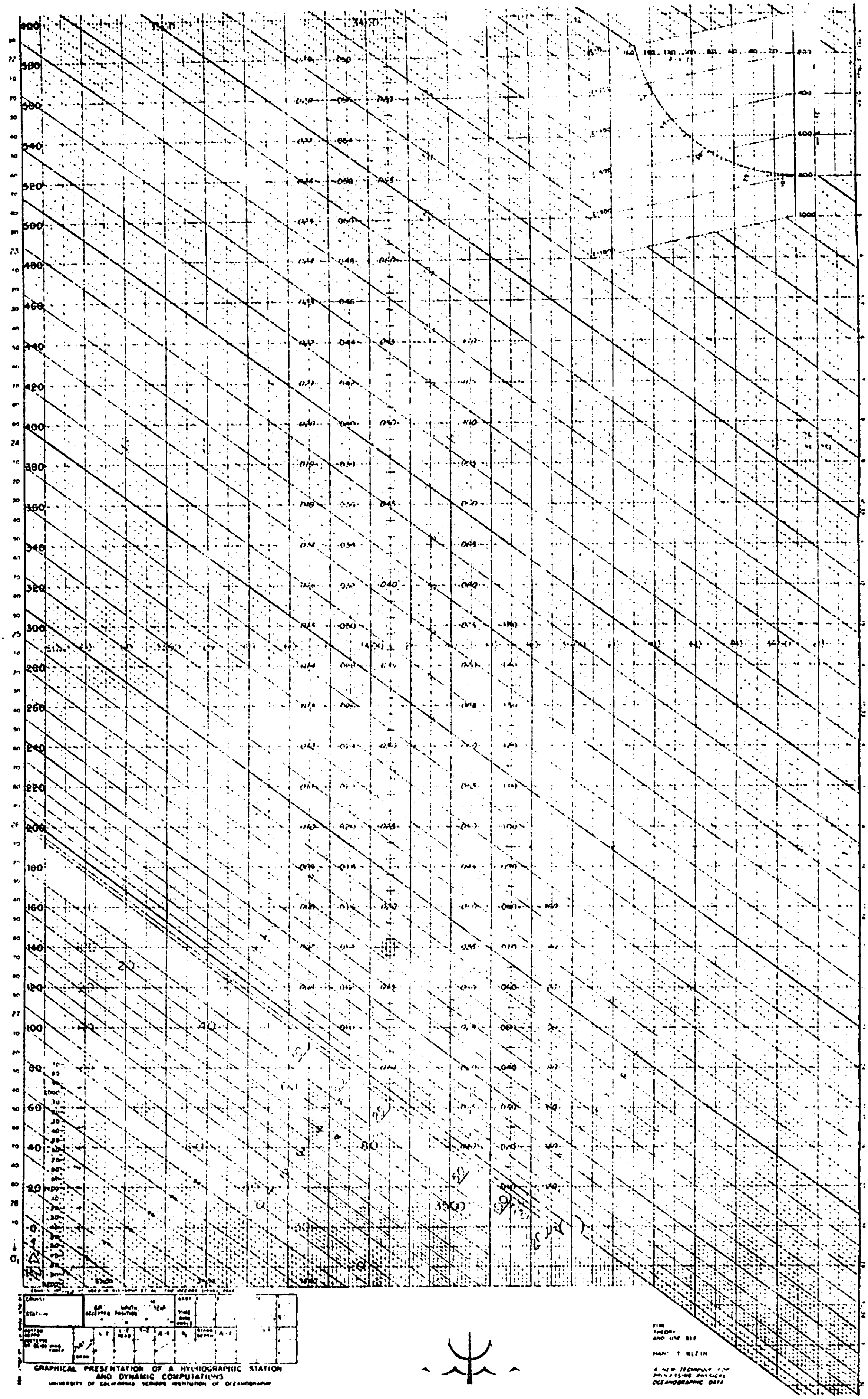
HANS T KLEIN
A NEW TECHNIQUE FOR PROCESSING PHYSICAL OCEANOGRAPHIC DATA



GRAPHICAL PRESENTATION OF A HYDROGRAPHIC STATION AND DYNAMIC COMPUTATIONS
UNIVERSITY OF CALIFORNIA, SCRIPPS INSTITUTION OF OCEANOGRAPHY

FOR THEORY AND USE SEE
HANS T. KLEIN
A NEW TECHNIQUE FOR PROCESSING PHYSICAL OCEANOGRAPHIC DATA

OCEANOGRAPHIC DATA SUMMARY

UNIVERSITY OF CALIFORNIA
SCRIPPS INSTITUTION OF OCEANOGRAPHY

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CCOFI
6407

60.190
STATION

VESSEL	STATION	MONTH	DAY	YEAR	HOUR (GCT)	LATITUDE °N	LONGITUDE °W	SOUNDING (F)	WIND DIR	WIND FORCE	BAROMETER	TEMPERATURE °F DRY BULB	WET BULB	WEATHER	CLOUDS TYPE	AMT.	SEA	SWELL DIR	HT. FT.	PERIOD	WIRE ∠
~~HORIZON~~ A. AGASSIZ	60.190	7	6	1964	1530	33 17	132 42.5	2670	010	3											07 13

OBSERVED

Z (DEPTH) (METERS)	T (°C)	S (‰)	O_2 (ml/L)	PHO	SIL		δ_T ($10^{-5}cm^3/g$)
0	18.80	33.695	5.75	0.34	2		
10	18.78	33.689	5.88	0.30	2		
30	17.34	33.698	5.90	0.30	2		
60	15.06	33.658	5.75	0.27	2		
80	14.98	33.725	6.09	0.21	2		
94	15.32	33.892	6.02	0.23	2		
108	15.66	34.075	5.92	0.20	2		
132	16.04	34.317	5.85	0.17	2		
152	16.27	34.426	5.70	0.18	3		
176	12.47	33.803	5.45	0.65	6		
205	10.61	33.794	5.29	1.00	11		
268	8.98	33.966	4.88	1.44	21		
340	7.96	33.992	4.07	1.79	32		
436	6.38	33.986	2.93	2.45	51		
532	5.44	34.066	1.57	2.94	72		
628	4.58	34.137	1.05	3.17	90		
748	4.19	34.259	.55	3.39	104		
871	3.98	34.363	.56	3.43	113		
1018	3.60	34.439	.58	3.46	122		
1161A	3.28	34.490	.90	3.33	127		
1167	3.29	34.492	.85	3.37	130		
1369A	2.88	34.535	1.14	3.24	132		
1610A	2.46	34.579	1.45	3.05	146		
1859A	2.13	34.577	1.47	3.19	147		

INTERPOLATED / COMPUTED

Z (DEPTH) (METERS)	T (°C)	S (‰)	O_2 (ml/L)			σ_t (mg/cm^3)	δ_T ($10^{-5}cm^3/g$)	ΔD (DYN M)	
0									
10									
20									
30									
50									
75									
100									
125									
150									
200									
250									
300									
400									
500									
600									
700									
800									
1000									
1200									
1500									
2000									
2500									
3000									
4000									

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OCEANOGRAPHIC DATA SUMMARY

UNIVERSITY OF CALIFORNIA
SCRIPPS INSTITUTION OF OCEANOGRAPHY

CRUISE CCO 64..

STATION 60..

VESSEL ~~HORIZON~~ A. AGASSIZ | STATION | MONTH | DAY | YEAR 19 | HOUR (GCT) | LATITUDE °N | LONGITUDE °W | SOUNDING (F) | WIND DIR. FORCE | BAROMETER | TEMPERATURE °F DRY BULB WET BULB | WEATHER | CLOUDS TYPE AMT. | SEA | SWELL DIR. HT. FT. PERIOD S. | WIRE ∠

OBSERVED

Z (DEPTH) (METERS)	T (°C)	S (‰)	O_2 (ml/L)	PHO	SIL		δ_T ($10^{-5}cm^3/g$)
2106A	1.89	34.633	2.12	3.00	15.7		
2353A	1.79	34.651	2.41	2.95	157		
2579A	1.68	34.649	2.58	2.88	159		
2797A	1.64	34.666	2.73	2.73	160		
2994A	1.59	34.673	2.96	2.80	159		
3191A	1.55	34.687	3.18	2.80	159		
3388A	1.52	34.680	3.27	2.75	156		
3586A	1.49	34.680	3.36	2.76	153		
3782A	1.50	34.684	3.42	2.74	150		
3929A	1.52	34.684	3.46	2.73	152		
4076A	1.51	34.687	3.52	2.72	149		
4272A	1.52	34.687	3.60	2.70	147		
4469A	1.53	34.690	3.66	2.69	144		
4665A	1.56	34.691	3.66	2.71	142		
4864A	1.58	34.690	3.71	2.70	143		
4963A	1.60	34.689	3.75	2.67	144		

INTERPOLATED / COMPUTED

Z (DEPTH) (METERS)	T (°C)	S (‰)	O_2 (ml/L)			σ_t (mg/cm³)	δ_T ($10^{-5}cm^3/g$)	ΔD (DYN M)
0								
10								
20								
30								
50								
75								
100								
125								
150								
200								
250								
300								
400								
500								
600								
700								
800								
1000								
1200								
1500								
2000								
2500								
3000								
4000								

OCEANOGRAPHIC DATA SUMMARY

UNIVERSITY OF CALIFORNIA
SCRIPPS INSTITUTION OF OCEANOGRAPHY

1/1

CRUISE: CCO 64 — STATION: 63.

VESSEL	STATION	MONTH	DAY	YEAR	HOUR (GCT)	LATITUDE °N	LONGITUDE °W	SOUNDING (F)	WIND DIR	WIND FORCE	BAROMETER	TEMPERATURE °F	WEATHER	CLOUDS	SEA	SWELL	WIRE ∠
~~HORIZON~~ A. AGASSIZ	63.200	7	7	1964	0333	32 21.5	133 00	2500	320	3							

OBSERVED

Z (meters)	T (°C)	S (‰)	O_2 (ml/L)	PHO.	SIL.		δ_T ($10^{-5}cm^3/g$)
0	19.20	33.845	5.74	0.57	2		
9	18.90	33.834	5.87	0.50	2		
28	17.71	33.852	5.86	0.46	2		
56	16.02	33.794	5.70	0.42	2		
73	15.78	33.845	6.03	0.41	2		
86	15.76	33.860	6.00	0.35	2		
100	15.74	33.883	5.93	0.30	2		
124	15.84	34.024	5.84	0.35	2		
141	15.50	34.039	5.70	0.37	3		
162	15.99	34.298	5.66	0.35	3		
199	13.92	33.995	5.46	0.51	5		
247	9.80	33.818	4.87	1.34	15		
313	8.92	33.982	4.33	1.66	33		
399	7.34	34.015	3.11	2.29	41		
490	6.18	34.070	1.65	2.91	60		
578	5.40	34.124	1.01	3.68	73		
690	4.91	34.243	.56	3.36	87		
803	4.46	34.331	.43	3.41	97		
943	3.95	34.407	.52	3.45	110		
1087	3.67	34.480	.82	3.41	115		
1175A	3.38	34.503	.96	3.71	121		
1373A	2.96	34.546	1.18	3.33	132		
1622A	2.56	34.577	1.48	3.10	141		
1869A	2.19	34.523	1.06	3.26	125		

INTERPOLATED / **COMPUTED**

Z (meters)	T (°C)	S (‰)	O_2 (ml/L)				σ_t (mg/cm³)	δ_T ($10^{-5}cm^3/g$)	ΔD (dyn m)
0									
10									
20									
30									
50									
75									
100									
125									
150									
200									
250									
300									
400									
500									
600									
700									
800									
1000									
1200									
1500									
2000									
2500									
3000									
4000									

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OCEANOGRAPHIC DATA SUMMARY

UNIVERSITY OF CALIFORNIA
SCRIPPS INSTITUTION OF OCEANOGRAPHY

CRUISE: CCOF. 640[illegible]

63.2[illegible]

STATION

VESSEL: ~~HORIZON~~ A. AGASSIZ

STATION | MONTH | DAY | YEAR 19 | HOUR (GCT) | LATITUDE °N | LONGITUDE °W | SOUNDING (F) | WIND DIR. FORCE | BAROMETER | TEMPERATURE °F DRY BULB, WET BULB | WEATHER | CLOUDS TYPE, AMT. | SEA | SWELL DIR. HT. PERIOD | WIRE ∠

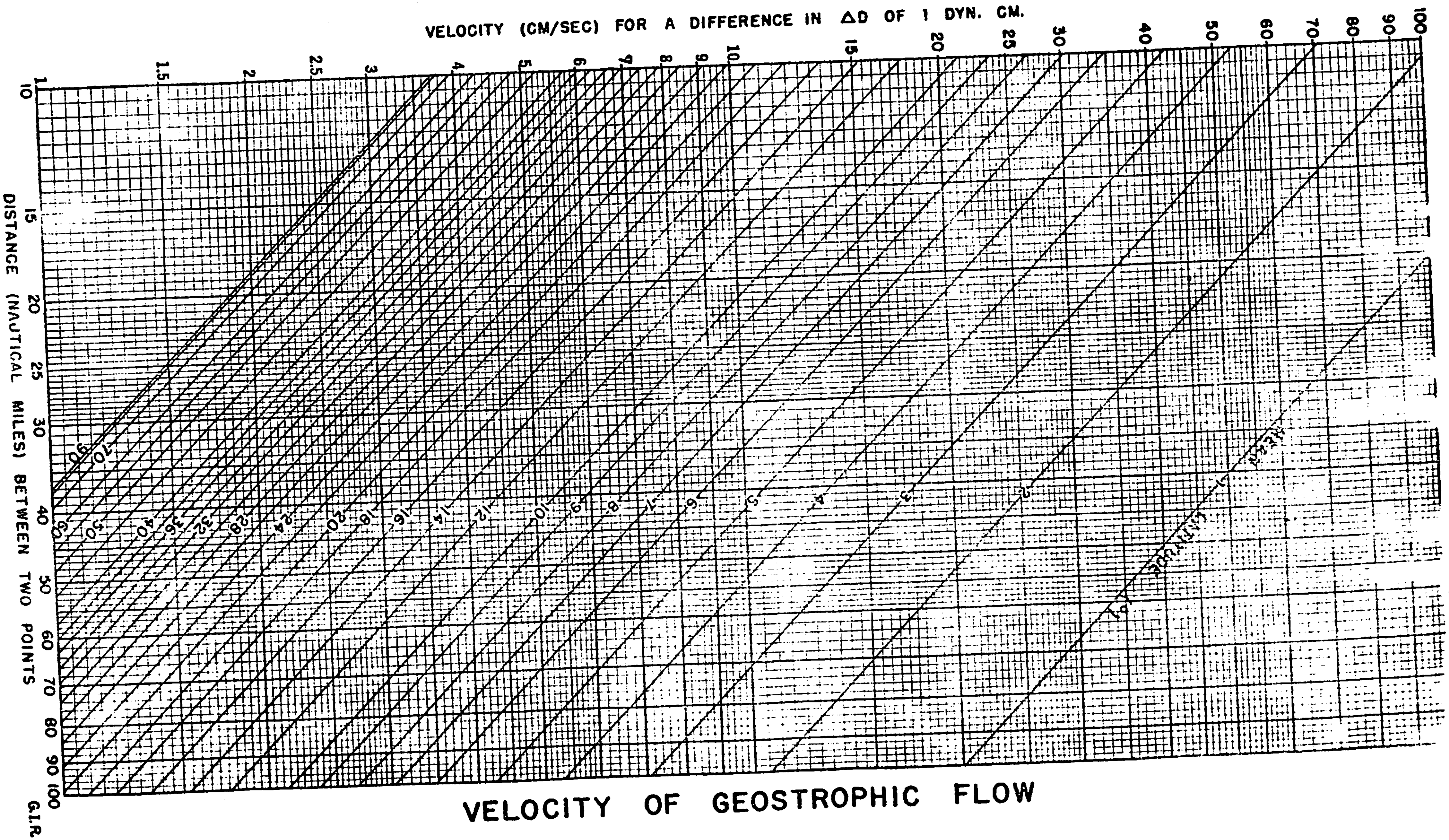
OBSERVED

Z (DEPTH) (METERS)	T (°C)	S (‰)	O_2 (ml/L)	PHO.	SIL.		δ_T (10^{-5} cm³/g)
2116A	1.96	34.626	2.00	3.01	156		
23[illegible]A	1.86	34.640	2.14	2.93	162		
2510A	1.74	34.640	2.28	2.99	156		
2707A	1.67	34.657	2.56	2.84	162		
2903A	1.65	34.666	2.73	2.78	160		
3100A	1.58	34.673	2.92	2.76	162		
3297A	1.54	34.676	3.09	2.75	159		
3492A	1.50	34.676	3.24	2.71	154		
3688A	1.51	34.687	3.29	2.72	156		
3883A	1.51	34.685	3.44	2.75	152		
4029A	1.52	34.690	3.47	2.76	149		
4175A	1.52	34.689	3.42	2.68	150		
4370A	1.55	34.689	3.50	2.68	149		
4564A	1.58	34.684	3.59	2.68	147		
4757A	1.60	34.690	3.54	2.69	147		
4854A	1.60	—	3.47	3.28	188		

INTERPOLATED / COMPUTED

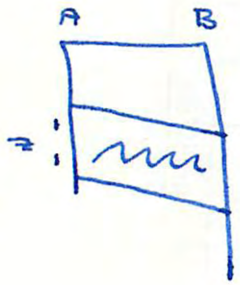
Z (DEPTH) (METERS)	T (°C)	S (‰)	O_2 (ml/L)				σ_t (mg/cm³)	δ_T (10^{-5} cm³/g)	ΔD (DYN M)
0						0			
10						10			
20						20			
30						30			
50						50			
75						75			
100						100			
125						125			
150						150			
200						200			
250						250			
300						300			
400						400			
500						500			
600						600			
700						700			
800						800			
1000						1000			
1200						1200			
1500						1500			
2000						2000			
2500						2500			
3000						3000			
4000						4000			

LA JOLLA AND POINT LOMA, 1953



VELOCITY OF GEOSTROPHIC FLOW

Discharge = $\overset{\text{Area}}{\overbrace{L \times W}} \times$ Velocity



$$Q = \underset{\text{Fixed}}{AB} \int \partial c \int \partial z$$

$$= AB \int \underset{\text{Assume Avg. velocity}}{\bar{c}}\, \partial z$$

As $\quad c = \frac{1}{f AB} \left[D_B - D_A \right]$

$$Q = \frac{1}{f} \int D_B - D_A \, \partial z$$

Q is reported in Sverdrups = 10^6 cu meters/sec

* understand units for dynamic depth

" dept of no motion

H. H. equation

CE 201 A Physical Oceanology
Ocean Engineering

Department of Civil Engineering
Div. of Hydraulic & Sanitary Engr.

EARTH SCALE FLUID CIRCULATION

Before one can discuss the general circulation of the oceans, the interactions of the two earth encircling fluids air and water, and the mechanics of the major driving force solar radiation must be considered. As the atmosphere overlies the ocean and is less dense and thus is more fluid than sea water, the atmosphere shows the influence of solar radiation more readily.

Characteristics of the sun"s radiation

The maximum wave length of the sun's radiation is 0.6μ which is in the short or ultra violet band. Very little of the ultra violet below 0.4μ reaches the earth as this wave length is absorbed by oxygen and ozone in the upper atmosphere. The λmax of the sun is essentially that of a black body radiating at 5800°K. The atmosphere, however, is reasonably transparent to visible and near infrared wave lengths so that most of the solar energy that reaches the earth's surface is in the band 0.3μ to 3.0μ. Any radiation longer than 3.0μ is absorbed by atmospheric water and that from 14 to 16μ is absorbed by CO_2. As noted above radiation below 0.4μ is absorbed by oxygen and ozone.

The radiation that strikes the earth warms it up. However to maintain a heat balance the earth must return to space an amount equal to that received. Because of the absorption effects of the atmosphere the earth can

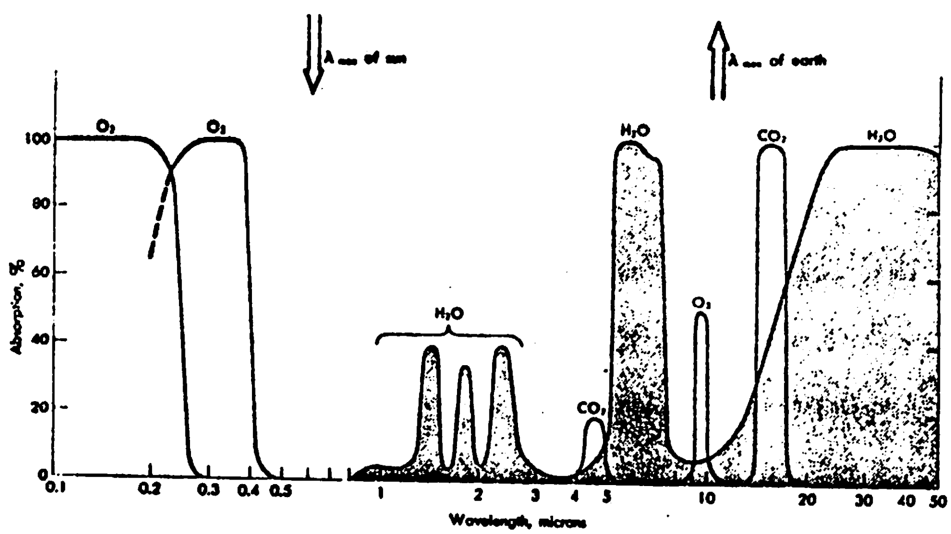


Fig. 6-5. Schematic diagram of the absorptivity of water vapor and other atmospheric gases. [Generalized from R. M. Goody and G. D. Robinson, 1951, *Quart. J. Roy. Met. Soc.*, 75.]

Von Arx, W. S., 1962, AN INTRODUCTION TO PHYSICAL OCEANOGRAPHY: Reading, Mass., Addison-Wesley, p. 145.

only reradiate in a narrow band from 8 to 12μ. Thus the temperature of the earth adjusts to permit reradiation at this wave length. This is the "green house" effect which gives the earth a somewhat higher temperature than its distance from the sun warrents.

The unit of solar radiation is the Langley which is 1 calorie/cm^2. The practical energy unit is Langleys/minute

$$Ly/min = 700 \text{ watts/meter}^2$$
$$\sim 1.0 \text{ horsepower/meter}^2.$$

The solar constant at the outer limits of the earth's atmosphere at right angles to the earth is 2 gram ly/min. However the average input is 0.5 gram ly/min as all radiation does not strike the earth at right angles (see Lave and Drummond, 1968, p. 838).

Heat Distribution

The amount of isolation that reaches the surface of the earth and is absorbed is a function of season (angle of tilt of earth's axis) latitude, time of day (angle of sun), and the local albedo. The albedo is a parameter of the reflectivity of the earth and will vary with cloud, vegetative, snow, and ice cover. On the average the zonal (latitudinal) albedo is:

(1) Low at the equator ? - much radiation is absorbed by high cloud cover as this is a zone of mostly water. (cloudy at Equator)

(2) Relatively high at mid latitudes 30-40°, maximum at 30°. Clear air in desert regions (horse latitudes).

ADVECTIVE PROCESSES

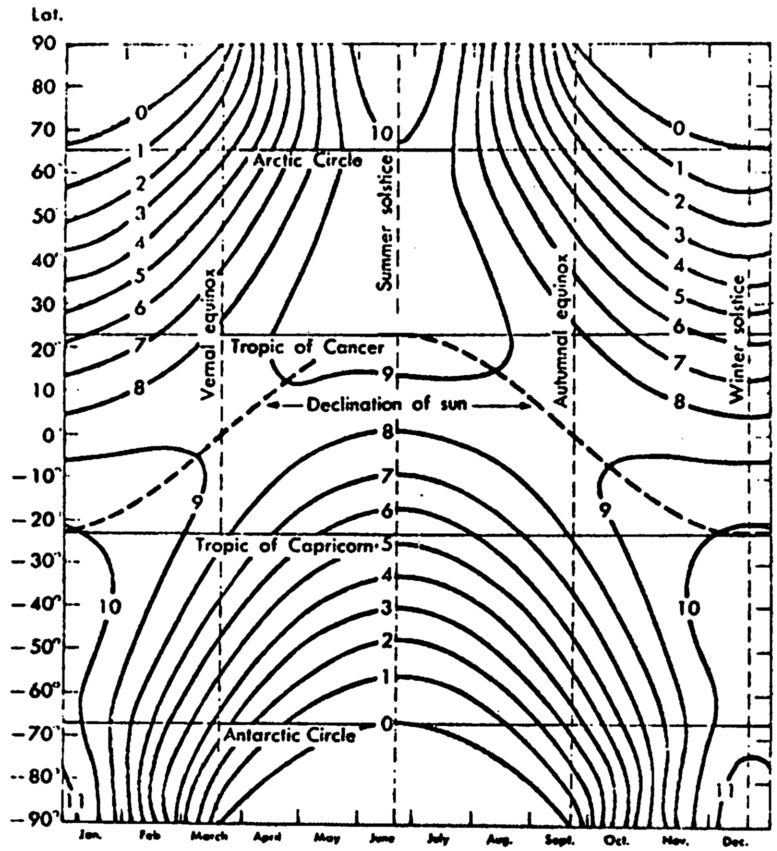


Fig. 6-2. Undepleted insolation, in hundreds of langleys per day, as a function of latitude and date (after List). Cross-hatched areas represent latitudes within the earth's shadow. [From Smithsonian Meteorological Tables, 6th edition, Washington, D.C.: The Smithsonian Institution.]

Von Arx, W. S. , 1962, AN INTRODUCTION TO PHYSICAL OCEANOGRAPHY: REading. Mass., Addison-Wesley, p. 142.

(3) Lower until polar regions

(4) High at poles because of relatively high reflectivity of snow and ice. - reflectivity

As there is more land in the northern hemisphere the net effect is that the northern hemisphere is 1°K cooler than the southern hemisphere (287°K vs 288°K mean surface termperature) thus there must be a trans geographic equatorial heat transport northward so the meterological equator is actually about 5° north of the geographical equator..

In general between latitudes 38°N and 38°S more heat is absorbed than radiated. At higher latitudes more heat is radiated than is absorbed. This produces a poleward heat flux. Heat demand is reduced somewhat as latitudinal surface areas decrease polarward. The average isolation to the earth's surface is about 0.25 ly/min or about one half of the input at the top of the atmosphere.

Atmospheric circulation

The atmosphere appears to be the major agency for polarward heat transfer because of the combination of (1) the atmospheric absorption of about half the solar input, (2) the more rapid movement of air with respect to sea water, and (3) the large amounts of latent heat carried by water vapor, for example 1.0 inch of rain by condensation releases latent heat at 25 ly/min or 100 times that of the average solar input to the ground under clear skies.

As could be assumed from the above discussion the primary cause of atmospheric circulation is differential heating. In the tropics, especially along the equator which receives the maximum long term insolation because of

n o

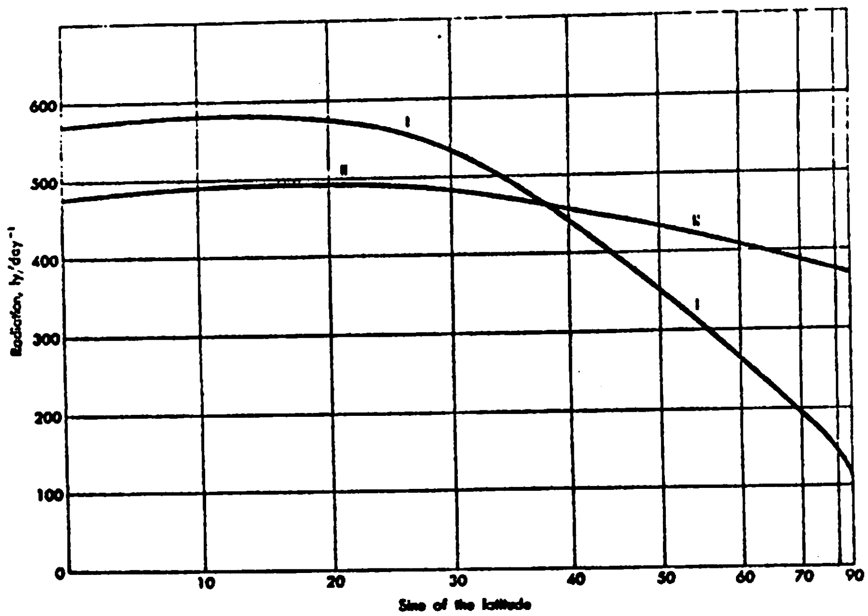
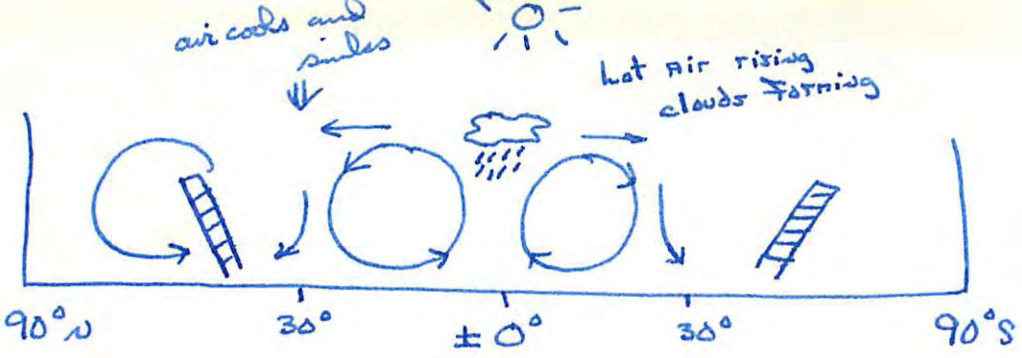


Fig. 6-6. Annual mean solar radiation absorbed by earth and atmosphere (curve I) and outgoing long-wave radiation leaving atmosphere (curve II) as functions of latitude. [From H. G. Houghton, 1954, *J. Met.*, 11(1).]

Von Arx, W. A., 1962, AN INTRODUCTION TO PHYSICAL OCEANOGRAPHY: Reading, Mass., Addison-Wesley, p. 148.



meterological equator shifts with the season, around the geographical equator and will result in the air masses being subjected to the Coriolis effect - flux of heat towards poles with a W ⇒ E motion in N. Hemisphere from Coriolis

{ wind direction from where it comes
{ current " where it is going

30° air sinks as it cools and is caught in up draft resulting in south movement in N hem. - deflected towards west from Coriol. - called east trade winds

called the Hadley cell or circulation

very dry at 30°

at poles air sinks because of cold and flows south with deflection towards west in N. hemisphere

T.C. - thermal contrast - when cold air meets warm air and cold air slides underneath

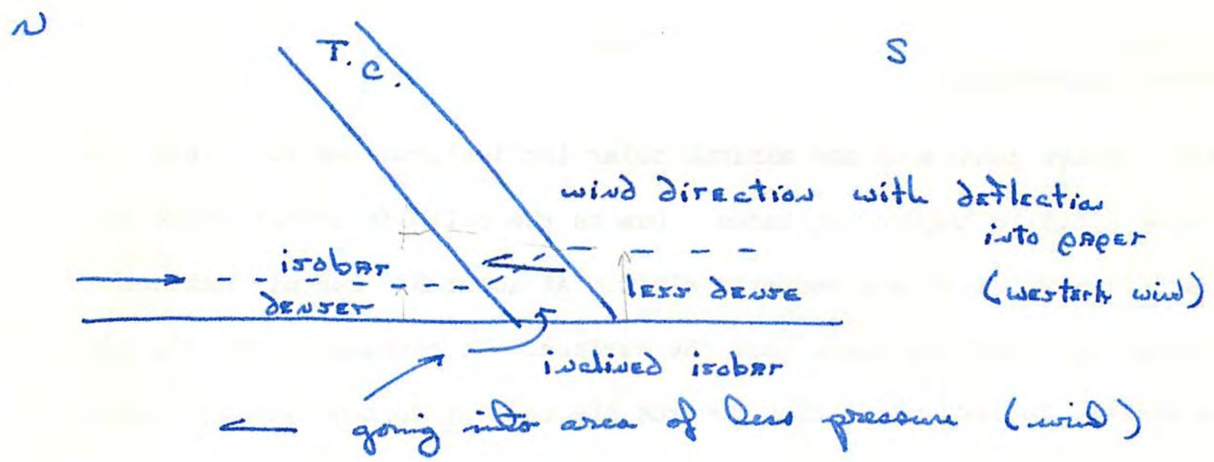
"jet stream" is at thermal contrast

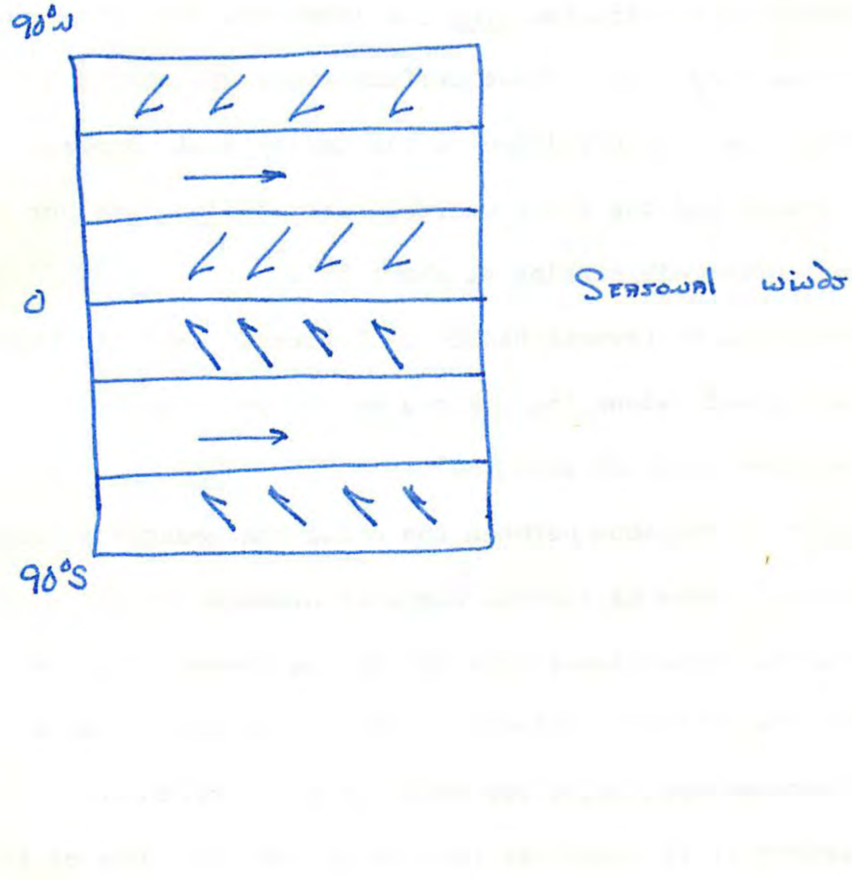
the greater zonal area and minimum solar inclination, hot air rises and flows aloft to higher latitudes. Due to the coliolis effect these polar-ward flowing winds are westerly aloft. At about 30° the air has cooled enough to bring the winds near the surface. To compensate for the equatorial updrafts, surface winds flow towards the equator to complete the cell. Such winds flowing equatorward are deflected to the west producing easterly winds (winds are considered to be directed from the direction they flow, currents towards the direction they flow. These surface winds are the trade winds. The tropical circulation can be generalized as the Hadley cell composed of the surface easterly trades and the aloft westerly anti-trades with hot air rising at the equator and cooled air sinking at about 30°.

At the poles a type of reverse Hadley cell forms. Cold air sinks over the poles and spreads along the surface as the polar easterlies with an updraft at the polar front at about 60°.

The circulation in the zone between the polar and equatorial cells is controlled by the sharp zone of thermal contrast produced by the shift from a thermal excess at latitudes lower than 38° to the thermal deficits in higher latitudes. The zone of contrast is shifted to higher latitudes at higher altitudes because cold polar air sinks under warmer tropical air contains much moisture as it overrides the cooler air, the zone of thermal contrast is a belt of persistant cloudness moving between 30° and 50°.

Ferrel attempted, before the discovery of the thermal contrast and the jet stream, to construct the cell-like circulation for the mid-latitudes. However the westerlies in this belt are not as persistant as the trades in


N
T.C.
S
wind direction with deflection
into paper
(westerly wind)
isobar
denser
less dense
inclined isobar
← going into area of less pressure (wind)


90°N
0
90°S
Seasonal winds

the Hadley cell. The best explanation for the temperate zone westerlies is that the thermal contrast tends to support a horizontal pressure gradient across the zone of contrast because the pressure falls off more slowly with altitude in warm air than in cold air. Thus a higher pressure is found at higher altitudes in the tropics than in the temperate regions. This produces a polarward flow of air which is deflected to the east by the Coriolis effect and forms the temperate westerlies.

CE 201A Ocean Engineering
Physical Oceanology

Department of Civil Engineering
Division of Hydraulic & Sanitary Engr.
Instructor: P. Wilde

test up to here

NON-FRICTIONAL COUPLING BETWEEN THE ATMOSPHERE AND OCEANS

Ignoring for the moment the effects of winds on oceanic circulation, let us consider just the thermal effects of the atmospheric blanket covering the oceans. As noted above this is represented by

(1) the green house effect, and

(2) the polarward transfer of heat most likely as latent heat in the atmospheric moisture.

The obvious effects on the oceans are

(1) to modify the surface temperature, and

(2) monitor the sea surface salinity by

(a) evaporation when H_2O is given up to the atmosphere and

(b) when H_2O is added to the oceans from the atmosphere.

The surface waters of the oceans are divided into three climatic subdivisions: (1) equatorial water, (2) central water of the mid-latitudes and (3) polar water. The temperature of these waters obviously increase towards the equator. However the salinity as shown by Wust is a function of the local balance of evaporation and precipitation. Precipitation has a trimodal distribution with the primary mode at the meterological equator caused by high rainfall produced from condensation of moist tropical air in the equatorial updraft of the Hadley cell. Two secondary modes occur at mid-latitudes produced by precipitation at the thermal contrast.

The evaporative distribution is Bl modal with two modes in the horse latitudes about 20° to 30° where descending cold air from the Hadley cell picks up moisture from the oceans and also produces desert conditions at these latitudes on land. High rates of evaporation are caused here both by

(1) deficient moisture content of cold descending air and

(2) the generally clear skies in the horse latitudes which permit greater insolation.

Low salinity at 90° and 0° because of ice melting
and rain fall (surface salinity - Dense saline water
at poles, sinks)

surface conditions are conditions that affect circulation

The combination of these latitudinal distrubutions produce

(1) two salinity maxima in the horse latitudes and

(2) a minimum near the equator. Salinity decreases polarward from the two maxima.

The generalized picture of surface circulation (Again ignoring frictional wind produced effects) is the warm fresher water at the meterological equator spreads polarward chiefly horizontally. At the horse latitudes the high salinity here produces sinking so the surface layer is thickest there. However there is still a significant polarward horizontal movement. This more saline water rides over the much colder although fresher (more dense) water formed near the poles.

Large scale variations in temperature and salinity can be produced only at the surface because

(1) the heat flow from the terrestrial earth through the sea floor is on the order of 10^{-6}ly/sec or 10^{-4} less than that of solar insolation,

(2) except for very restricted regions at very shallow depths no appreciable dissolved solids are lost to the oceans at depth.

In essence the oceans are undersaturated with respect to the major constituents on a world-side basis.

Chemical evidence such as

(1) the constancy of the ratio of the major constituents of sea water and

(2) the presence of dissolved oxygen, which could only come from the atmosphere, below the depth of photosynthetic plants all suggest a well mixed ocean.

Thus the primary driving force of mixing must be gravitational produced by density differences initiated by variations of temperature and salinity at the surface, the only place where significant amounts of heat at salt can be exchanged. Therefore primary water masses are formed at the surface and the character of the water mass depends on (1) latitude, (2) oceanographic climate, (3) degree of isolation, and (4) degree of mixing by winds, currents (ignored for the present).

Except for localized small water masses as are found in the Mediterranean and Red Seas, the densest open ocean waters are formed in the polar regions. In this case temperature is the major factor effecting density.

As the Antarctic regions are open equatorward into the three world oceans; the Pacific, Atlantic, and Indian the formation of water masses here are of great significance. Arctic formed waters are of secondary importance volumetrically because the Arctic is essentially a closed basin and much of the water formed in the Arctic Ocean is trapped behind by shallow sills leading into the Pacific and the Atlantic Oceans.

93.3% of ocean water is colder than 10°C and 76.5 °/oo is colder than 4°C thus the obvious conclusion is that most ocean water has its characteristics determined primarily in the polar regions and spreads equatorward driven by its higher density.

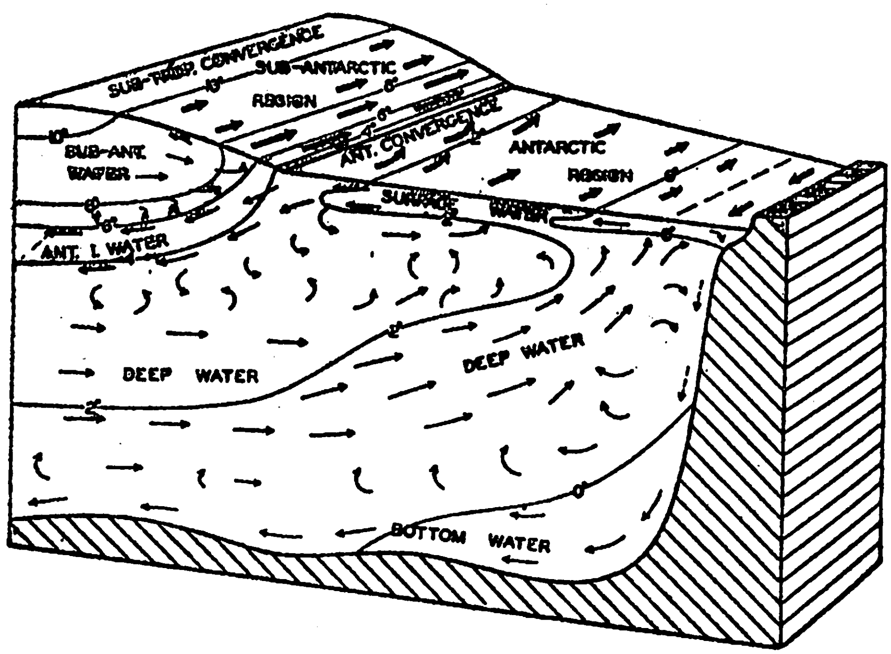
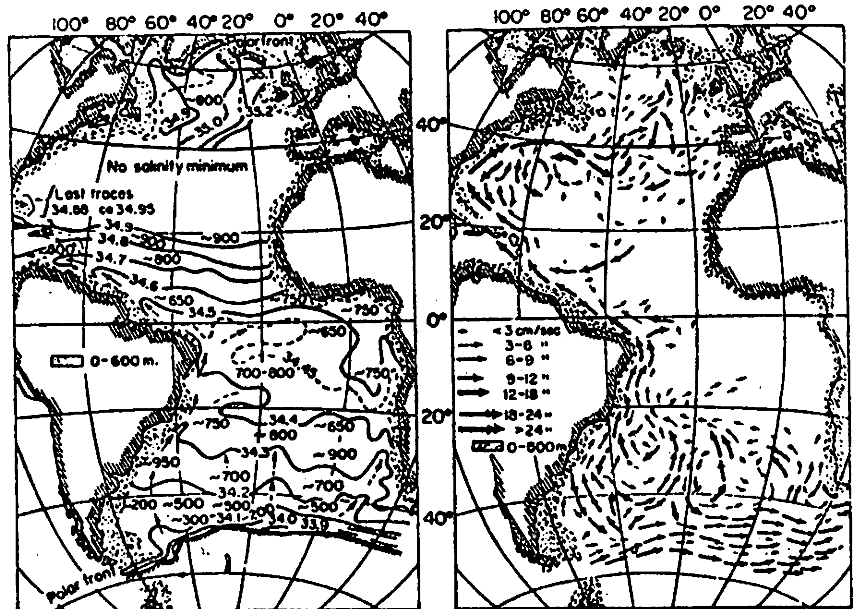


Fig. 164. Schematic representation of the currents and water masses of the Antarctic regions and of the distribution of temperature.

Sverdrup, H. V., Johnson, M. W., and Fleming, R. H., 1942, THE OCEANS: Englewood Cliffs, N. J., Prentice-Hall, p. 620.



(a) (b)

Fig. 197. Spreading of water masses of Subantarctic and Subarctic Intermediate Water in the Atlantic Ocean. *a*, Represents the distribution of salinity in the layer of salinity minimum (at approximately 500-900 m.) Added figures: Depth distribution of this layer (according to G. Wüst, 1936); *b*, represents the current field at an 800-m. depth (according to A. Defant, 1941).

Dietrich, G., 1963, GENERAL OCEANOGRAPHY: New York, Interscience-John Wiley, p. 482.

Dietrich, G., 1963, GENERAL OCEANOGRAPHY:
New York, Interscience, John Wiley,
p. 484-485.

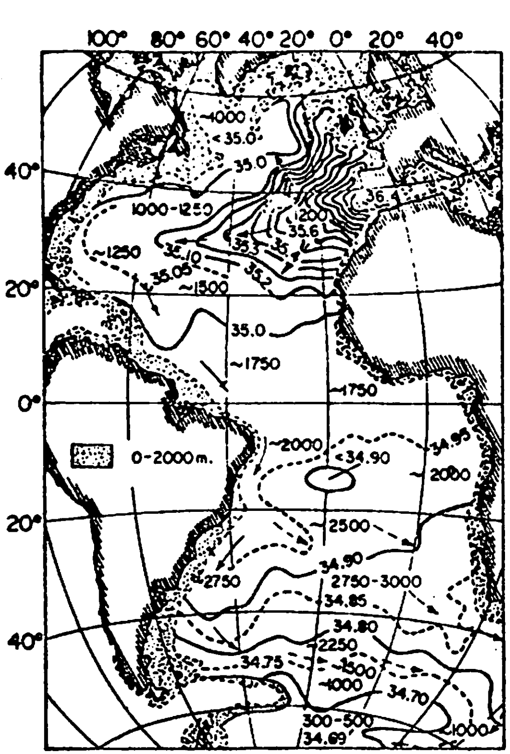


Fig. 198. Spreading of water masses in the upper North Atlantic Deep Water, represented by the distribution of salinity in the layer of maximum salinity (at approximately 1000-3000 m. depth). Added figures: Depth distribution of this layer in m. (according to G. Wüst, 1936.)

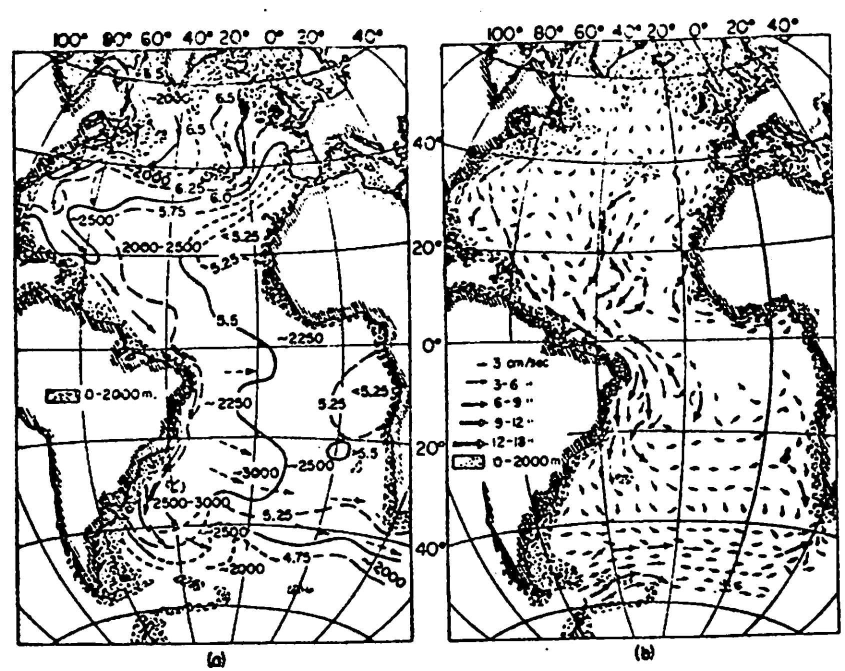


Fig. 199. Spreading of water masses in the middle North Atlantic Deep Water in the Atlantic Ocean. *a*, Represents the distribution of oxygen in the layer of maximum oxygen (at approximately 2000-3000 m. depth). Added figures: Depth distribution of this layer in m. (according to G. Wüst, 1936). *b*, Represents the current field at 2000 m. depth (according to A. Defant, 1941).

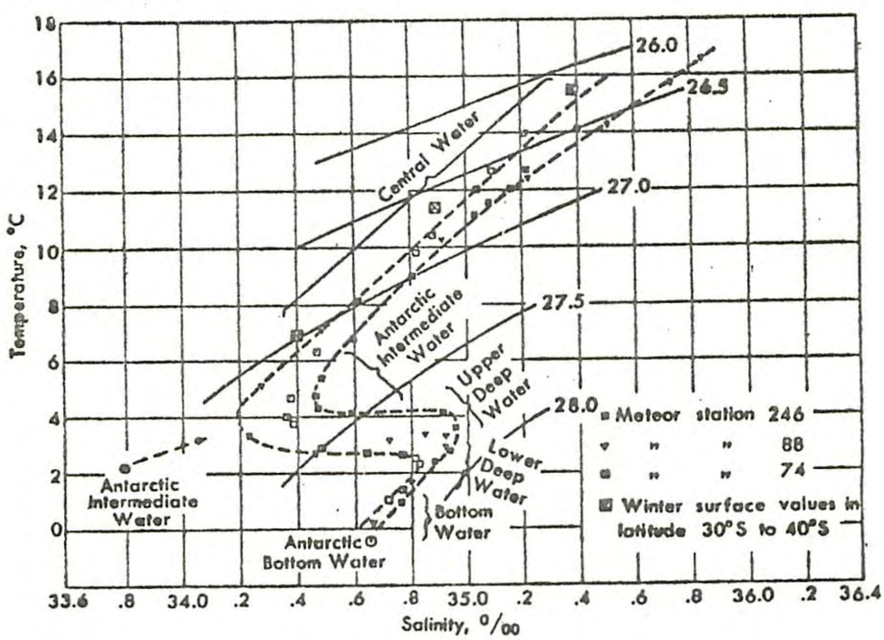


Fig. 7–11. Temperature and salinity of the water masses of the Atlantic Ocean, as derived from *Meteor* stations 74, 88, and 246. [After H. U. Sverdrup, M. W. Johnson, and R. H. Fleming, 1942, *The Oceans, Their Physics, Chemistry, and General Biology*, New York: Prentice-Hall.]

meterological equator

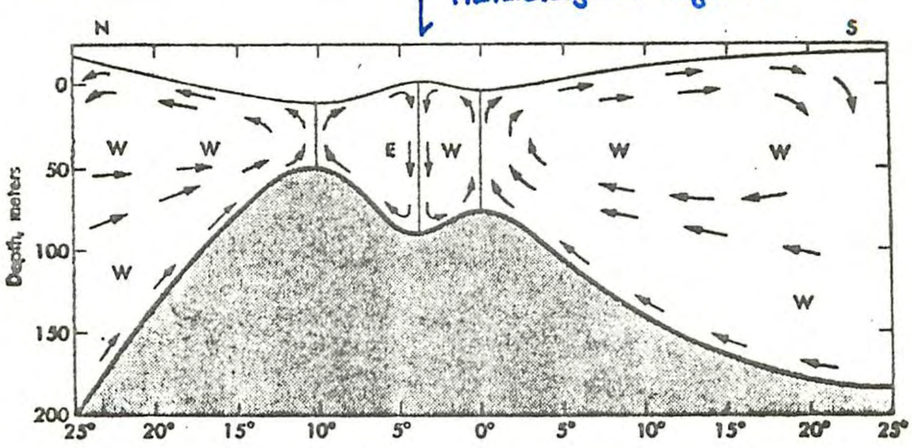


Fig. 7–12. Schematic representation of the vertical circulation within the equatorial region of the Atlantic. The main direction of the currents is indicated by the letters W and E. The water below the discontinuity surface, which is considered to be at rest, is shaded. (After Defant), [adapted from H. U. Sverdrup, 1942, *Oceanography for Meteorologists*, New York: Prentice-Hall.]

Von Arx, W. S., 1962, INTRODUCTION TO PHYSICAL OCEANOGRAPHY. Reading, Mass., Addison-Wesley, p. 200.

Fig. 196. Areas of formation (⊙) and spreading of Antarctic and Arctic Bottom Water in the Deep-Sea Basins (> 4000 m.) of the world ocean, represented by the distribution of the potential temperature t_p at the bottom. (Following G. Wüst, 1938.)

Dietrich, G., 1963, GENERAL OCEANOGRAPHY: New York, Interscience-John Wiley, p. 479.

TABLE LXIII

Temperature, Salinity, and Oxygen Content near the Bottom in the Basins and Trenches of the Austral-Asiatic Mediterranean[a]

Number[b]	Name	Sill depth, m.	Greatest depth, m.	Potential temp., deg.	S ‰	Oxygen content[c]
I	Sulu Basin	400	5,580	9.84	34.47	8.9
II	Philippine Trench	—	10,500	1.16	34.67	44.5
III	Talaud Trough	3,130	3,450			
IV	Sangihe Trough	2,050	3,820	2.15	34.65	35.7
V	Celebes Basin	1,400	6,220	3.26	34.59	28.7
VI	Morotai Basin	2,340	3,890	1.55	34.68	39.5
VII	Ternate Trough	2,710	3,450	1.65	34.66	39.3
VIII	Batjan Basin	2,550	4,810	1.78	34.66	34.1
IX	Mangole Basin	2,710	3,510			
X	Gorontalo Basin	2,700	4,180	1.95	34.65	
XI	Makassar Trough	2,300	2,540	3.39	34.51	27.4
XII	Halmahera Basin	700	2,039	7.54	34.60	43.2
XIII	Buru Basin	1,880	5,319	2.66	34.63	33.8
XIV	Northern Banda Basin	3,130	5,800	2.73	34.60	32.6
XV	Southern Banda Basin	3,130	5,400	2.75	34.62	32.6
XVI	Weber Deep	3,130	7,440	2.75	34.63	31.6
XVII	Manipa Basin	3,100	4,360	2.85	34.60	32.1
XVIII	Ambalau Basin	3,130	5,330	2.75	34.61	33.4
XIX	Aru Basin	1,480	3,680	3.62	34.62	22.6
XX	Butung Trough	3,130	4,180			
XXI	Salajar Trough	1,350	3,370	3.66	34.60	28.9
XXII	Flores Basin	2,450	5,130	2.96	34.60	30.4
XXIII	Bali Basin	—	1,590	3.46	34.61	26.9
XXIV	Savu Basin	2,100	3,470	3.14	34.56	27.1
XXV	Wetar Basin	2,400	3,460	2.92	34.61	31.1
XXVI	Timor Trench	1,940	3,310	2.57	34.68	34.2
XXVII	Sunda Trench	—	7,140	0.77	34.67	59.5

[a] According to P. M. van Riel (1934, 1943, 1950).
[b] See Figure 209.
[c] Per cent of saturation near the bottom.

Dietrich, G., 1963, GENERAL OCEANOGRAPHY: New York, Interscience-John Wiley, p. 502.

AIR-SEA INTERACTIONS
ADDITIONAL REFERENCES

NAMIAS, J., 1973, RESPONSE OF THE EQUATORIAL COUNTERCURRENT TO THE SUBTROPICAL ATMOSPHERE: SCIENCE, V. 181, P. 1244-1245

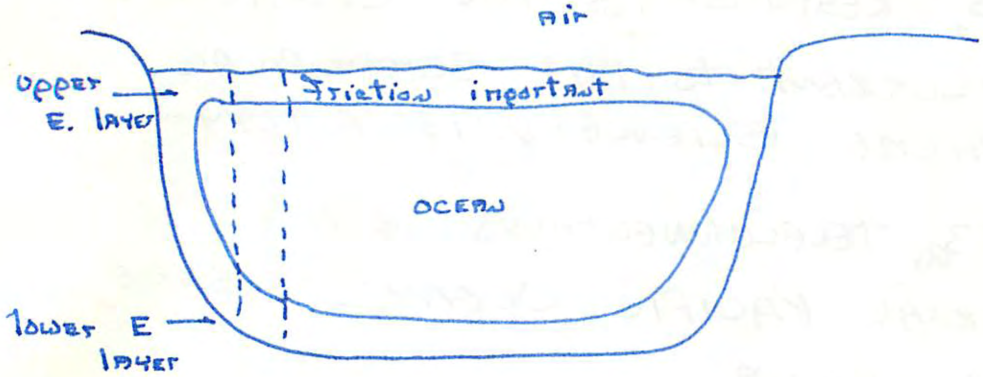
WYRTKI, K., 1973a, TELECONNECTIONS IN THE EQUATORIAL PACIFIC OCEAN: SCIENCE, V. 180, P. 66-68

WYRTKI, K, 1973 b, AN EQUATORIAL JET IN THE INDIAN OCEAN: SCIENCE, V. 181, P. 262-263

MALKUS, J.S., 1962, LARGE-SCALE (SEA-AIR) INTERACTIONS, IN HILL (Ed.) THE SEAS, V. I, PHYSICAL OCEANOGRAPHY: NEW YORK, INTERSCIENCE, P. 88-294.

ECKART, C., 1960, HYDRODYNAMICS OF OCEANS AND ATMOSPHERES: NEW YORK, LONDON, PERGAMON, 290P.

BJERKNES, J., 1966, A POSSIBLE RESPONSE OF ATMOSPHERIC HADLEY CIRCULATION TO EQUATORIAL ANOMALIES OF OCEAN TEMPERATURE: TELLUS, V. 18, NO. 4, P. 820-829



Air
upper E. layer
Friction important
OCEAN
lower E layer

Ocean Engineering
CE 201A
Physical Oceanography

Department of Civil Engineering
Division of Hydraulic & Sanitary Engr.
Instructor: P. Wilde

FRICTIONAL EFFECTS (not for test)

For large scale water motions simple laminar flow appears to be not realistic instead, motion of water in the oceans is characterized by numerous eddies of varying dimension which produce irregualr motion called turbulent flow. This motion can not be described by simple equations but is approximated by statistical methods. In this manner the velocity at any given point in space is the vector sum of

(1) $\bar{V}$, the long term average velocity and,

(2) V' the velocity due to turbulence.

So,

$$V_x = \bar{V}_x + V'_x$$

$$V_y = \bar{V}_y + V'_y$$

$$V_z = \bar{V}_z + V'_z$$

In essence turbulent flow implies a transfer of momentum from one fluid layer to another. The momentum must be proportional to:

(1) the difference between M of the original fluid and M where the exchange occurs or

$$-l_z \frac{d(\rho\bar{V}_x)}{dz}$$

and

(2) the vertical velocity of the fluid mass being exchanged or

$$|\bar{V}_z|'$$

So

$$M = -K\,|\bar{V}_z'|\,l_z \frac{d\rho\bar{V}_x}{dz}$$

where l_z = vertical distance of movement

$\rho\bar{V}_x$ = mass of fluid being exchanged

Defining l_z where $K = 1$ and assuming constant ρ for small motions

$$M = -\rho |\bar{V}_z'| \; l_z \frac{d\bar{V}_x}{d_z}$$

Or

$$M = -A_z \frac{d\bar{V}_z}{d_z}$$

Where

The eddy viscosity $= A_z \equiv \rho |\bar{V}_z'| l_z$

By analogy with laminar flow where

Laminar $\tau = \mu \frac{dV}{dN}$; μ = Dynamic Viscosity,

Turbulent $\tau = A\frac{dV}{dN}$; A = Eddy Viscosity $= \eta$

So

$$\tau_{xz} = A_z \frac{d\bar{V}_x}{dz} + \frac{\mu \, d\bar{V}_x}{dz}$$

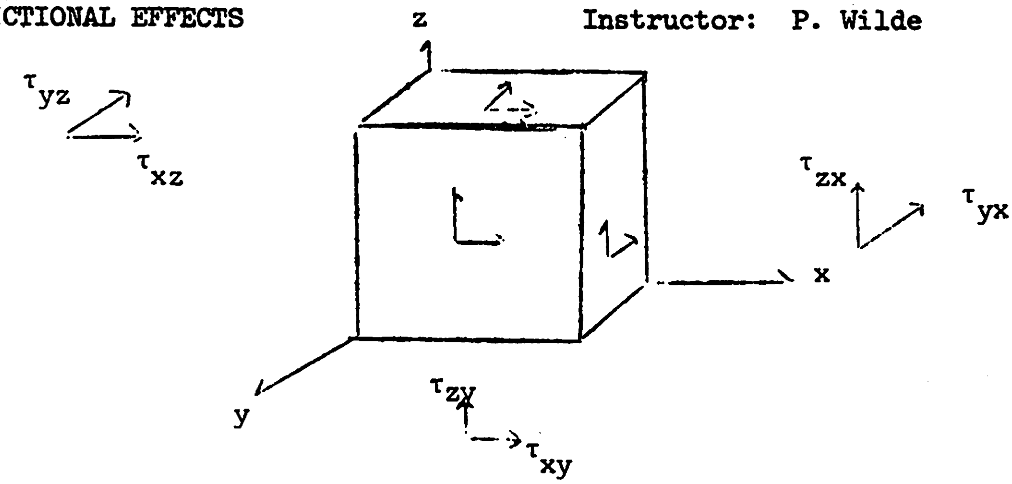
$$\tau_{yz} = A_y \frac{d\bar{V}_y}{dz} + \frac{\mu \, d\bar{V}_y}{dz}$$

As the eddy viscosity depends on the state of motion and is not a fundamental property of the fluid as is μ; "A" or η varies over wide limits (1 to 1×10^3 gm/cm/sec, Defant, 1961, p. 104) while μ is essentially constant for sea water (0.015). Also values of η are much larger than μ so the laminar term can be ignored. Thus

$$\tau_{xy} = \eta_y \frac{\partial \bar{V}_x}{\partial y} \qquad \tau_{yx} = \eta_x \frac{\partial \bar{V}_y}{\partial z}$$

$$\tau_{zy} = \eta_y \frac{\partial \bar{V}_z}{\partial y} \qquad \tau_{zx} = \eta_x \frac{\partial \bar{V}_z}{\partial x}$$

$$\tau_{xz} = \eta_z \frac{\partial \bar{V}_x}{\partial z} \qquad \tau_{yz} = \eta_z \frac{\partial \bar{V}_y}{\partial z}$$



$$R_x = \frac{\partial \tau_{xz}}{\partial_z} + \frac{\partial \tau_{xy}}{\partial y}$$

$$R_y = \frac{\partial \tau_{yz}}{\partial_z} + \frac{\partial \tau_{yx}}{\partial_x}$$

$$R_z = \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y}$$

Assume (1) only shear important

(2) $\bar{V}_z$ is small

(3) $\eta_x = \eta_y = \eta$ horizontal

Thus

$$R_x = \frac{\partial}{\partial z}\left(\eta_z \frac{\partial V_x}{\partial z}\right) + \frac{\partial}{\partial y}\left(\eta_H \frac{\partial V_x}{\partial y}\right)$$

$$R_y = \frac{\partial}{\partial z}\left(\eta_z \frac{\partial V_y}{\partial z}\right) + \frac{\partial}{\partial x}\left(\eta_H \frac{\partial V_y}{\partial x}\right)$$

$$R_z = 0$$

Frictional or Ekman Currents

As noted above the geostrophic assumption is no longer valid where frictional effects are important. This would be true

(1) at the sea surface where friction is produced by wind stress, and

(2) at the sea floor where currents would drag against the bottom.

Ekman (1905) in a classic series of papers derived mathematically the effects on currents produced by wind and bottom friction. As a result of these studies he proposed a three layer ocean model in which

UPPER EKMAN LAYER	Friction Important
GEOSTROPHIC LAYER	Pressure Gradient Acceleration Balanced by Coriolis Acceleration
LOWER EKMAN LAYER	Friction Important

The geostrophic layer where the current is initiated by a pressure gradient is overlain by

(a) a surface layer where frictional effects predominate and underlain by

(b) a bottom layer where frictional effects also predominate.

Ekman Solution for the Homogeneous Case

In the Ekman layers assuming that

(1) a steady state is attained so that the frictional effects balance the Coriolis acceleration;

(2) for homogeneous water or $d\rho = 0$;

(3) for horizontal isobars or

$$\frac{\partial P}{\partial x} = \frac{\partial P}{\partial y} = 0$$

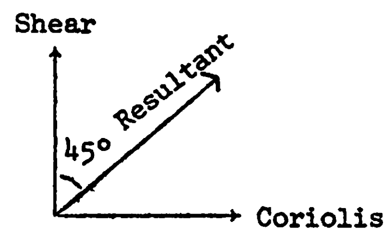
$$\rho f V_y + \frac{d}{dz}\left(A_z \frac{dV_x}{dz}\right) = 0$$

$$-\rho f V_x + \frac{d}{dz}\left(A_z \frac{dV_y}{dz}\right) = 0$$

These differential equations may be solved further assuming A_z is independant of depth. Generalizing

$$\rho f V + A\frac{\partial^2 V}{\partial z^2} = 0$$

In this case as the frictional effects are balanced by the Coriolis acceleration the resultant flow would be 45° cum sol (to the right in the Northern Hemisphere, or to the left in the Southern Hemisphere)



Northern Hemisphere
Upper Ekman Layer Case

As the frictional couple produced by the wind is applied at the sea surface its effects decay with depth or

$$V = V_o e^{-\frac{\pi Z}{D}}$$

Where V = Velocity at Depth Z
V_o = Velocity at $Z = 0$
$D = \pi\sqrt{\frac{A_Z}{\rho\omega\cancel{\sin\phi}}}$
$s\omega\phi$

where $Z = D$, $V = V_o e^{-\pi} = {}^{-1}/_{23}\, V_o = -0.043\, V_o$

at this depth V is directed π radians from V_o (180°) and for practical purposes frictional effect cease to have any importance. Where $Z = D$ is called the <u>depth of frictional resistance</u>.

D obviously is not only a function of the initial shear but also varies with latitude.

Empirical studies (see Dietrich, p. 343-345, 1963) indicate that

$$V_o = \frac{\lambda W}{(\mathrm{SIN}\ \phi)1/2}$$

Where

V_o = Surface Current Velocity

W = Wind Velocity

λ = 0.0126 if W in cm/sec

Approximately the wind drift velocity V_o = 1.5% of the wind velocity in moderate to high latitudes. Also numerically

$$D = \frac{7.6\ W}{(\mathrm{SIN}\ \phi)1/2}$$

For $\gamma^2 = 2.6 \times 10^{-3}$

$\rho_{\cdot air} = 1.25 \times 10^{-3}$

$\tau_a = 3.2 \times 10^{-6} W^2 g cm^{-1} sec^{-2}$

Where ω measured 15 meters above the sea surface.

Or numerically

$$D = 600\ V_o$$

For D in meters

V_o in cm/sec.

Munk (1947) considered 700 cm/sec to be the critical wind velocity

For W = 7 meters/sec

Latitude	SIN ϕ	$(\mathrm{SIN}\ \phi)^{1/2}$	D Meters
90°	1	1	53
80	.9848	.9923	54
70	.9397	.9694	55
60	.8660	.9306	57
50	.7660	.8752	61
40	.6428	.8017	66
30	.5000	.7107	75
20	.3420	.5848	91
10	.1736	.3166	168
0	0	0	---

However in the geologically interesting case, in the bottom Ekman layer the maximum shear is at the bottom and the reverse of the surface spiral obtains

$$V = V_g \, e^{\frac{\pi(Z-h)}{D}}$$

Where V_g = Velocity in overlying geostrophic layer

In other words the effects of friction increase as the bottom is approached and there is a backward (compared with the surface) or contra solem logrithmic decay downward (see diagrams). Thus it would be possible for geostrophic currents initially produced by pressure gradients in the intermediate geostrophic layer to induce bottom Ekman currents. The geologically preserved directional properties of such bottom Ekman currents would be a function of the hydraulic properties of

(a) existing bottom materials and

(b) material settling out the the geostrophic layer and

(c) the direction of the overlying geostrophic current.

The variation in azimuths of directional features ideally should be in the quadrant contra solem to the direction of the geostrophic current.

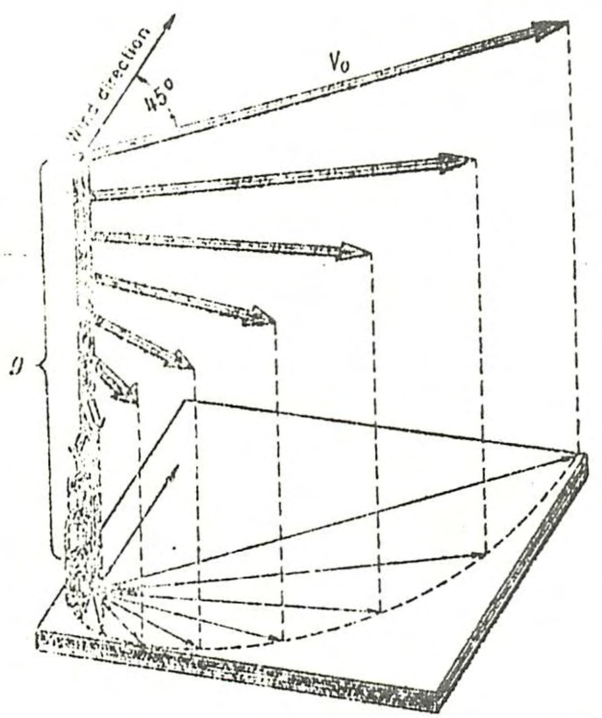


Fig. 143. Vertical current distribution in a pure drift current in the Northern Hemisphere. (According to V. W. Ekman, 1905.)

Log. Decay

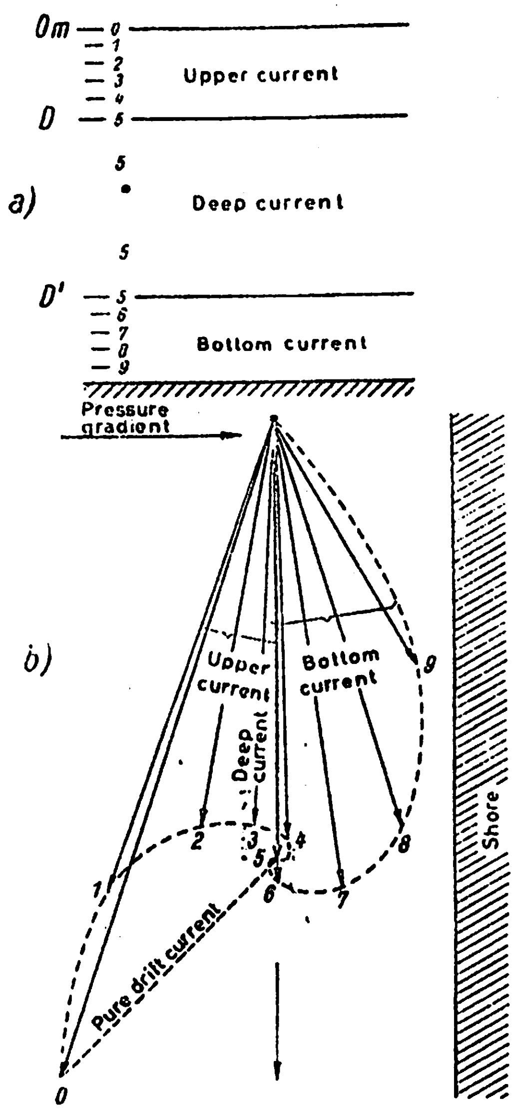


Fig. 145. Ekman's elementary current system.

do not influence each other by mixing. However, in nature, this assumption is fulfilled only in those rare cases where water masses are separated from each other by strongly developed discontinuity surfaces. In

REFERENCES

Ekman, V. W., 1905, On the Influence of the Earth's Rotation on Ocean Currents: Ark. Mat. Astrom. Dys., V. 2, No. 11, pa. 1-52.

Munk, W. H., 1947, A Critical Wind Speed for Air-Sea Boundary Processes: Jour. Marine Research, v. 6.

GENERAL

Dietrich, 1963, General Oceanography: Chapter VII, Theory of Ocean Currents.

Defant, 1961, Physical Oceanography: Chapter XIII, General Theory of Ocean Currents in a Homogeneous Sea.

Proudman, 1952, Dynamical Oceanography: Chapter IX, Internal Friction and Geostrophic Effects.

Sverdrup and others, 1942, The Oceans: Chapter XIII, Dynamics of Ocean Currents.

ADDENDA

HUNKINS, K., 1966, EKMAN DRIFT CURRENTS IN THE ARCTIC OCEAN: DEEP-SEA RESEARCH; V. 13, P. 607-620.

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OCEANOGRAPHIC CIRCULATION MODELS

The main aim in proposing models is to approach reality by considering as few variables as possible thus learning (1) what are the important parameters to study and (2) what time scale are these parameters useful. The following summary derived from Stommel (1957) discusses the development of useful oceanographic circulation models.

Hough (1897) proposed an oceanic model based on the assumptions that of (1) a limitless ocean (no land),(2) uniform depth, (3) no friction (4) precipitation in one hemisphere (northern) and evaporation in the other (southern). Thus the free water surface would be raised in the Northern Hemisphere and lowered in the Southern. This would produce a net transport to the Southern Hemisphere. The total effect would be high pressure in the hemisphere of precipitation and low pressure in the hemisphere of evaporation both pressure systems centered on the poles. Geostrophic flow would be zonal (parallel to latitudinal circles) although the net transport is always south. Unfortunately the mathematical Hough model accelerates so no steady exists.

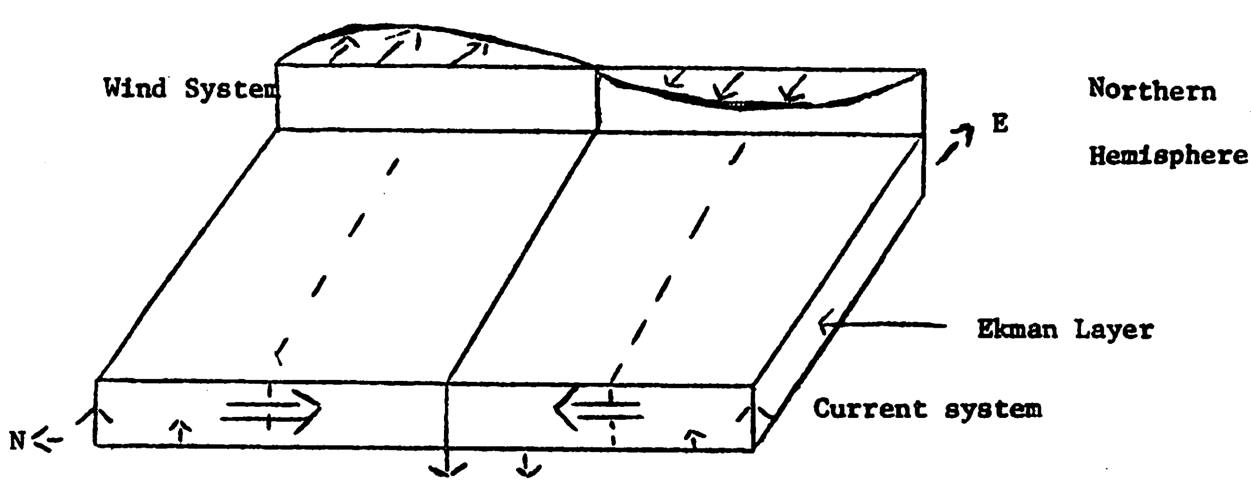
Goldsbrough (1933) improved on the Hough model by adding meridional boundaries and assuming that $\oint$ of precipition = 0 along each parallel between the meridional boundaries. This in effect rotates the Hough Model 90° so that the precapation - evaporation maxima lie on the equator. On the Eastern edge of the precipitation hemisphere are two low pressure systems oriented symmetrically about the equator. On the Western edge are two high pressure systems. Geostrophic flow will be both zonal(near the equator) and meridional (elsewhere). Between isobars in meridional flow water must be added when moving equatorward as the coriolis parameter decreases equatorward or

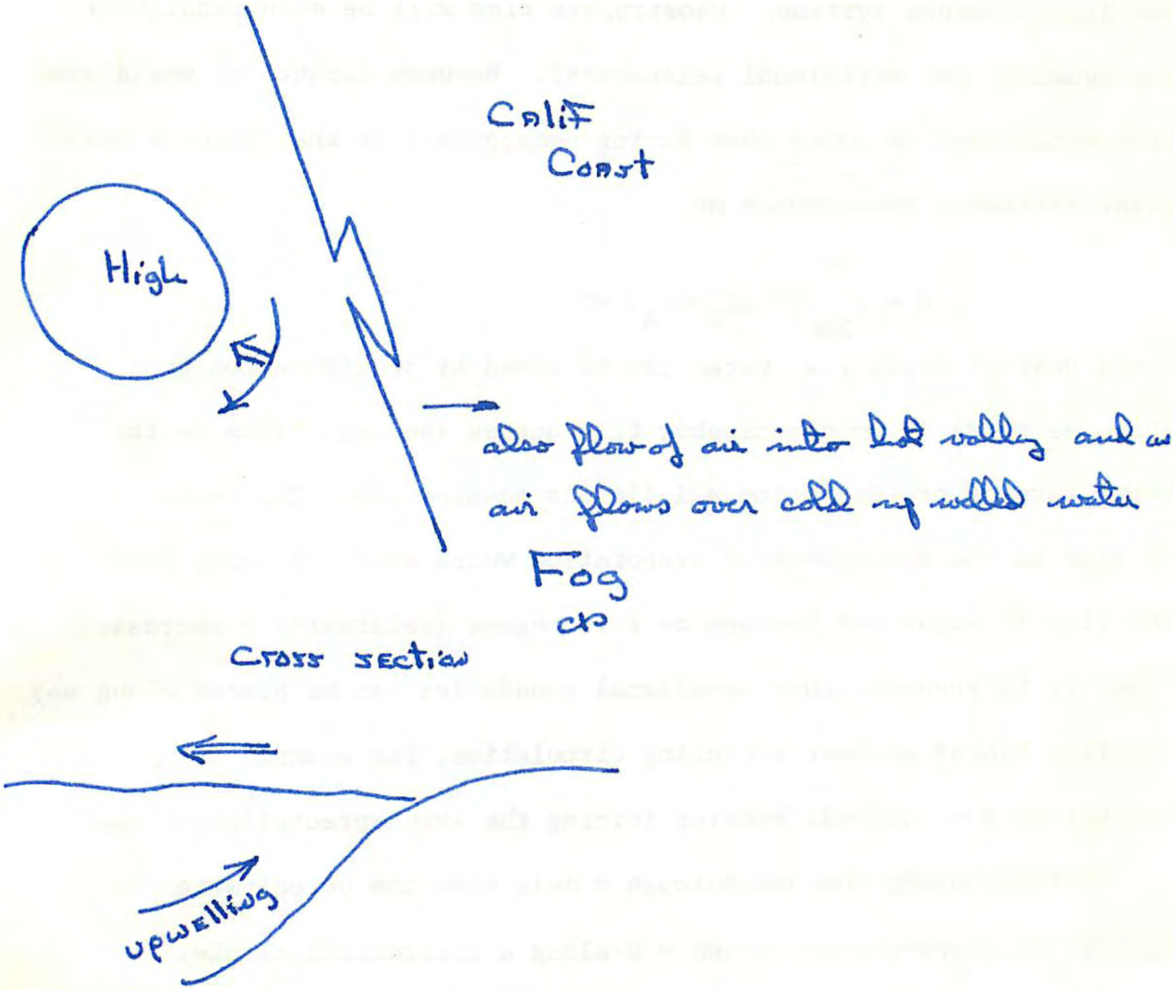
$$Q = \int_{Z_N}^{Z_o} \frac{1}{f} [\Delta D_B - \Delta D_A] \, dZ$$

Given uniform depth the water can be added by precipitation from above as there is no geostrophic flow across isobars. Thus in the hemisphere of precipitation all flow is equatorward. The reverse is true in the hemisphere of evaporation where water is being extracted so the flow is polarward because as f increases (polarward) Q decreases. Also, it is apparent that meridional boundaries can be placed along any complete isobar without affecting circulation, for example along the equator or a meridional barrier joining the evapo-precipitation maxima.

Unfortunately the Goldsbrough models have the unrealistic restriction that $\oint$ of evapo-precipitation = 0 along a latitudinal circle.

A possible way to produce oceanic pressure fields without resort to evapo-precipitation is by wind stress as proposed by Ekman (1905). A variable but zonal wind field (reasonable realistic) will produce convergence and divergence that initiate pressure changes in the underlying geostrophic ocean for a Northern Hemisphere model the wind.




Calif
Coast
High
also flow of air into hot valley and as
air flows over cold upwelled water
Fog
Cross section
upwelling

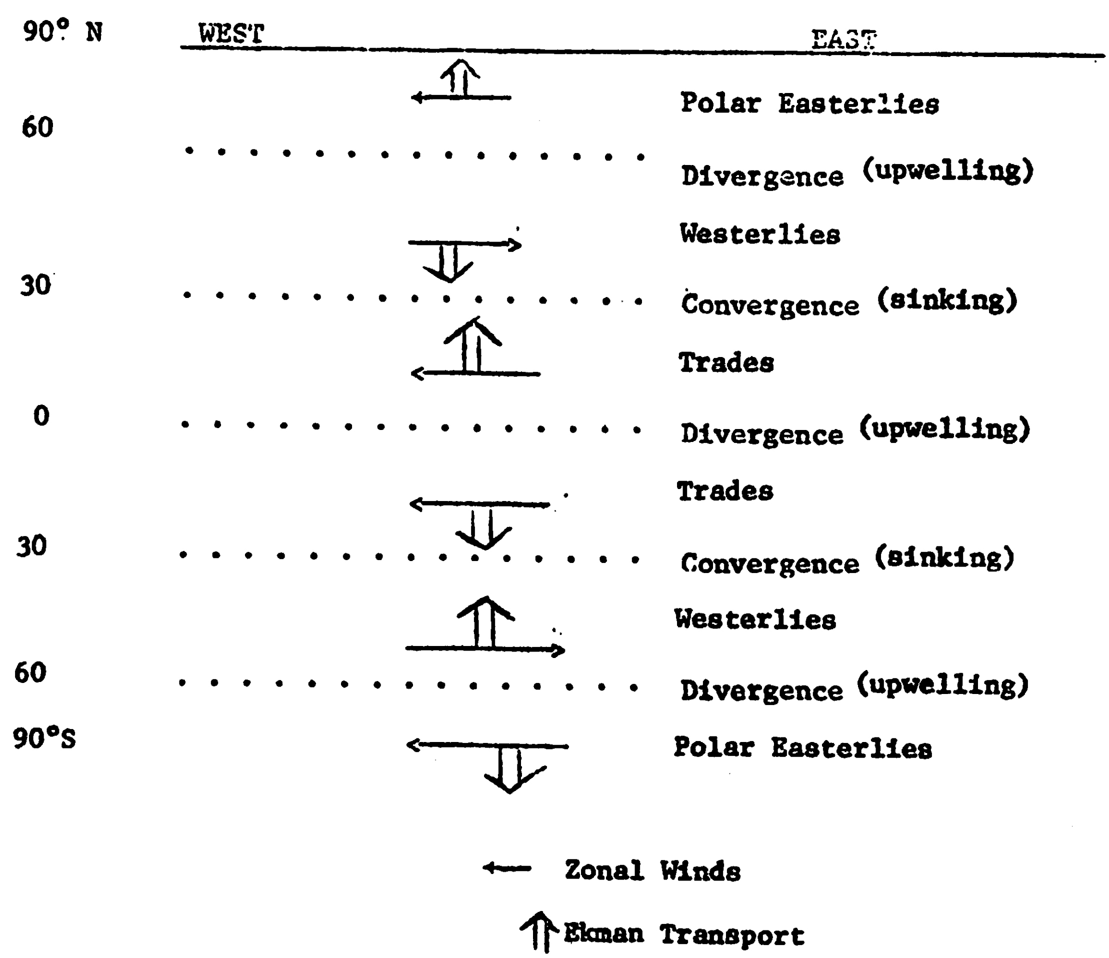
I. When the zonal wind shifts from west to east (to the south) at the node the Ekman transport shift from south to north so the transport vectors oppose each other and maximum sinking occurs (maximum high pressure).

II. When the zonal wind shifts from west to east (to the north) or from east to west (to the south) the Ekman transport vectors flow away from each other so up welling (Maximum low pressure) occurs.

Obviously from these arguments the node of vertical motion is where the zonal wind is at a maximum.

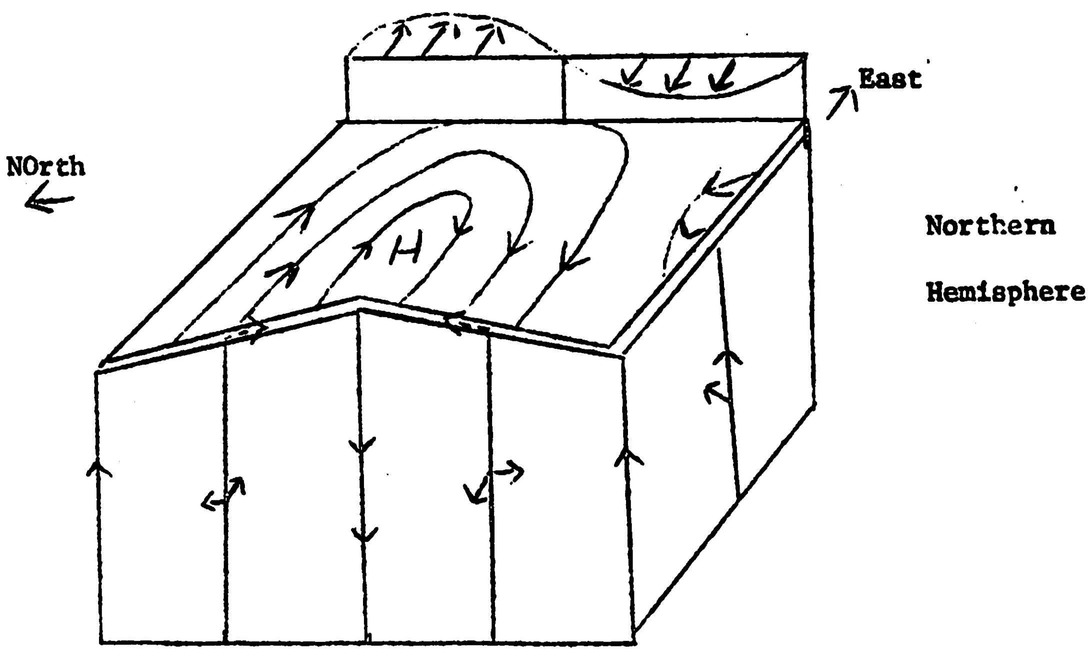
The effect on the underlying water in the geostrophic layer is (A) high pressure centered on Ekman convergences and (B) low pressure centered on Ekman divergences.

<u>EKMAN SURFACE LAYER CIRCULATION</u>



There are two possible ways to maintain continunity in the geostrophic layer beneath the Ekman surface layer (1) gradually reduce the vertical vector by meridional flow or (2) place a counter Ekman layer on the bottom which will have (a) divergency to carry away water sinking under surface convergence and (b) convergence upward to supply water for upwelling at surface layer divergences.

Sverdrup (1947) introduced an eastern boundary to the Ekman model:



In the Sverdrup model the flow is not completely zonal as there is significant meridional flow. The downward component vanishes below the initial zone of Ekman convergence. The excess water is absorbed by meridional transport away from the high pressure system. Along the zone of maximum winds there is no vertical component.

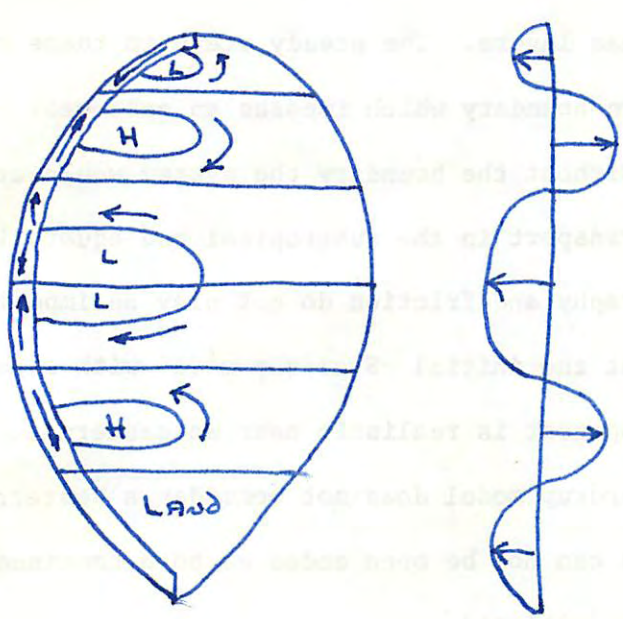
Addition of a bottom Ekman layer simply makes all geostrophic flow zonal in the sections away from the coast and places all compensating meridional flow in the Ekman layers. The steady state in these systems is produced by the eastern boundary which imposes an east-west pressure gradient on the system. Without the boundary the system would accelerate.

Investigations of transport in the subtropical and equatorial oceans suggest that bottom topography and friction do not play an important role in circulation so that the initial Sverdrup model with significant meridional geostrophic component is realistic near an eastern coast.

Unfortunately the Sverdrup model does not consider a Western coast. Obviously pressure systems can not be open ended as both continunity and pressure flow laws would be violated.

A way out of the western boundary problem was suggested by Carrier and Munk (1950). They realized that the introduction of an intense western boundary current at the western edge of the Sverdrup-Ekman model is the same as treating this area as the only region, outside of the Ekman layers, where friction is important or that elsewhere higher terms of friction and inertia may be ignored. This approach treats the intense western current as a boundary layer problem.

The addition of an intense western boundary current mades the Sverdrup-Ekman model more realistic as both the Gulf Stream and the Kuroshio current are intense western boundary currents.


L
H
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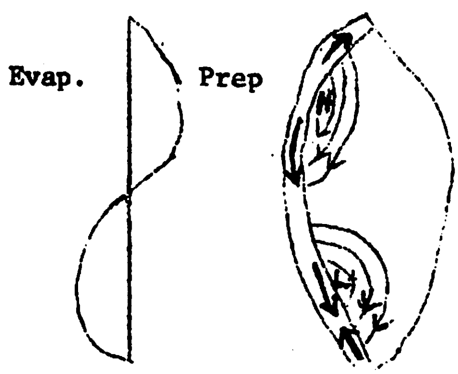
LAND

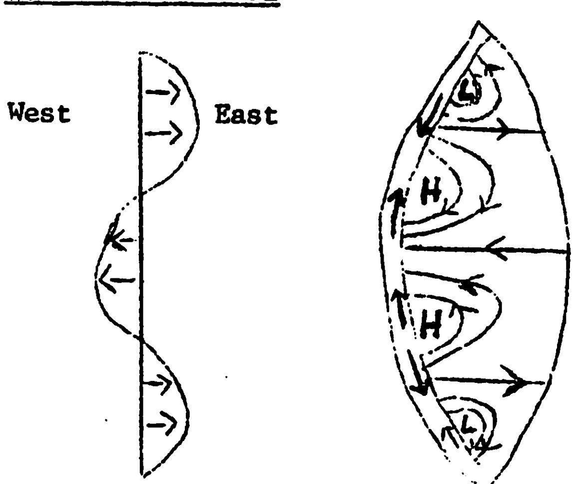
Also the western current may be used to satisfy continunity and mass conservation. Unfortunately the problem of the transition from low velocity geostrophic currents to intense frictional western currents has not been studied so for the purposes of the model the western current will be considered discrete and of a size sufficient to satisfy conservation of mass.

Hough Type Model



Notice that all transport across the equator is in the western boundary current as the geostrophic flow lines at the equator are zonal.

Wind Driven Model



No vertical motion

No vertical motion

No vertical motion

The Gyre boundaries are the latitudes of no Ekman convergence or where the geostrophic flow is zonal. The regions of maximum meridional geostrophic flow correspond to the latitude of maximum Ekman convergence (maximum equartorward geostrophic flow) and divergence (maximum polarward geostrophic flow). This model has been produced experimentally by Von Arx by applying wind stress over a circular rotating basin.

The parameter density has been ignored in the above models. As the oceans are density stratified it would seen advisable to introduce some kind of density change into the models. Reid (1948) Stockman (1953) and Lineykin (1955) have proposed various density models, although Lineykin's model shows the density changes produced by Ekman pile-up of water and demonstrates how friction produced changes can be transmitted into normal barotropic frictionless changes. This partially explains how the main thermocline develops at depths greater than $Z = D$ derived for wind drift currents. Partically density differences can be patterned by dividing up the model into just two layers and prescribe that vertical transport between the layers be a function of latitudé due to the change in the corious parameter and area to maintain an overall mass balance. In this manner the amount of water taken up or given up by each layer is governed by geostrophic flow and the western boundary current constructed to monitor the flow. Also the vertical transport between layers will vanish and the circulation will be governed by conditions with in the layer.

PROBLEMS OF STABILITY

The models studied so far are based on a steady-state pressure system over the oceans. As a first approximation particularily the Hough-Wind driven model resembles the mean pressure field at the ocean surface for the Atlantic Ocean. However, there is a valid question as to the significance of the mean picture and the resultant steady state with respect to reality. For example, the meteorologists believe that steady-state solutions do not apply to the atmosphere except possibly in the belt of trade winds.

Elsewhere, large quasi-geostrophic disturbances appear and disappear with high frequency which obviously have a great effect on the circulation of the atmosphere by analogy, as both air and water are fluids, the circulation of the oceans, except in the regions of the trade winds where the wind speed and direction is constant relatively, probably can not be related exactly to steady-state conditions. Stommel (1957) feels that, in general, the oceans may be represented by an approximation of steady-state conditions because the atmosphere, unlike the oceans, essentially is unrestricted by barriers and the only restrictions are lower boundary roughness produced by the configuration of the land and the sea surface.

For the oceans, which are much thinner than the atmosphere and confined in basins, the steady-state approximations of the Sverdrup model may be realistic, particularly because any high velocity zonal ocean currents are broken up by meridionally oriented coastlines. This line of reasoning suggests that the Atlantic and the Indian's circulation may be approximated by steady-state conditions as these oceans are essentially land locked. This would be less true for the Pacific, although some island chains and submarine ridges may approximate the effects of a coastal barrier. If the circulation of the Pacific is more like that of the atmosphere it becomes obvious why the numerous steady-state models proposed for the Pacific are less successful than are models proposed for the Atlantic.

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REFERENCE

Stommel, H., 1957, A survey of ocean current theory: Deep-Sea Research, V. 4., p. 149-184, (contains references cited above).

CE 201A Ocean Engineering
Physical Oceanology

Department of Civil Engineering
Division of Hydraulic & Sanitary Engr.
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OCEANIC CIRCULATION MODELS
PART II

RESPONSE OF OCEAN TO ATMOSPHERIC PRESSURE FIELD

Chase (1951) studied the variations in atmospheric pressure at the sea surface derived from weather records from about 1900-1950 for the North Atlantic. Here the generalized pattern is a high pressure cell approximately centered between 30° - 40° N. The center is at its north easterly most point in January. By March the center shifts almost 1,200 miles to the southwest at its most southerly point. From March to July the center moves due north, July to November due east until in November the center lies at the yearly average. The general appearance of the monthly pressure field is similar to the annual average. The westerlies are normally stronger in the winter. The circulation around the Bermuda high, in general, is greater in the summer which is caused by the relaxation of the intense low over the Northern Atlantic that forms in the winter as the Sub Arctic North Atlantic is much warmer than the surrounding continents. The result is the wind directions in the summer are steadier than in the winter. The velocities of the westerlies are decreased because of the northward shift of the high and the decrease in intensity of the Sub Arctic oceanic low. The summer velocities are lower than the violent winter westerlies in the North Atlantic.

The problem of frequency of occurrence is a major one, as only 1/2 of the daily weather maps resemble the monthly maps. In the summer there is a reasonable correspondence of about 60% as compared to 30% in the winter. The rest of the maps show various disturbances such as fronts, stagnant lows, linkages of the Bermuda high to continental highs, hurricanes, and deflection of the high center by low pressure areas.

The fundamental problem, of course, is the response time of the ocean. For models of the Barotropic Ocean (isopycnals and isobars parallel) the mean annual or even the monthly maps probably obtain - however for the upper surface, in the upper Ekman layer, for the Baroclinic Ocean the mean maps have no validity.

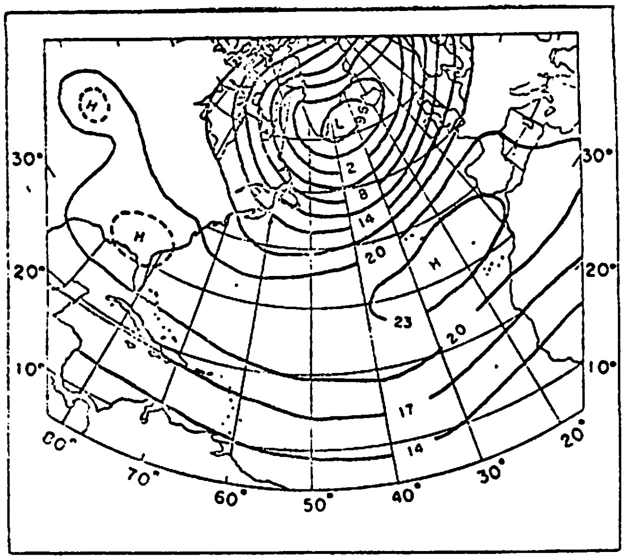


Fig. 47. Map showing mean sea-level pressure for January (Chase, 1951, MS). Pressures are given in the excess of millibars over 1000 mb.

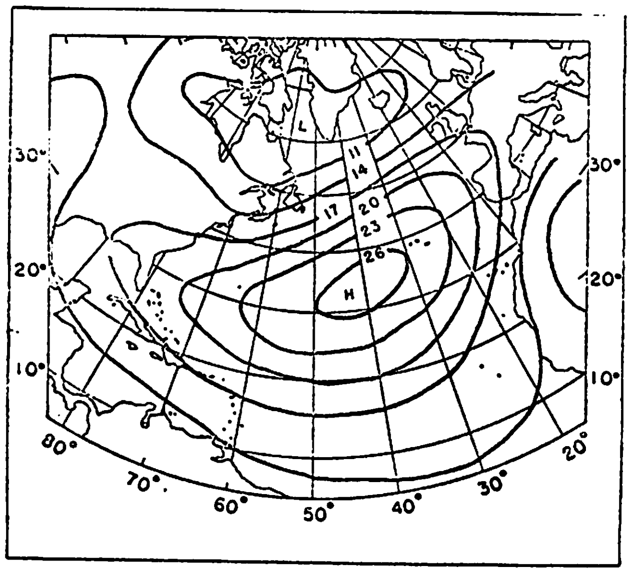


Fig. 48. Map showing mean sea-level pressure for July (Chase, 1951, MS). Pressures are given in the excess of millibars over 1000 mb.

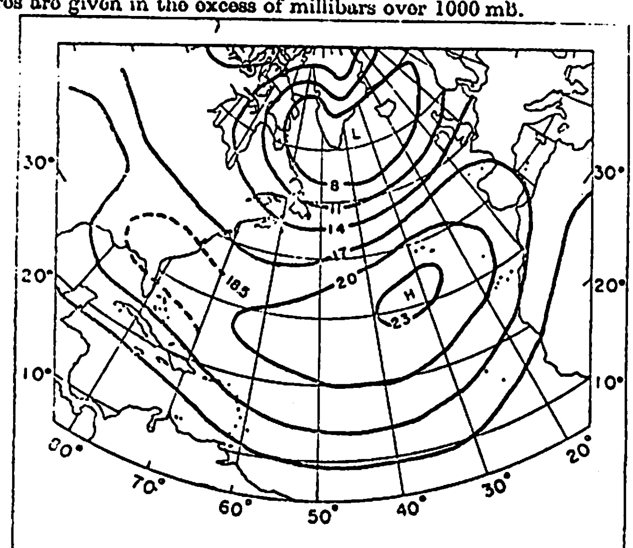


Fig. 49. Map showing annual mean sea-level pressure, according to Joseph Chase (1951, MS). Pressures are given in the excess of millibars over 1000 mb.

From Stommel (1960, p.76-77)

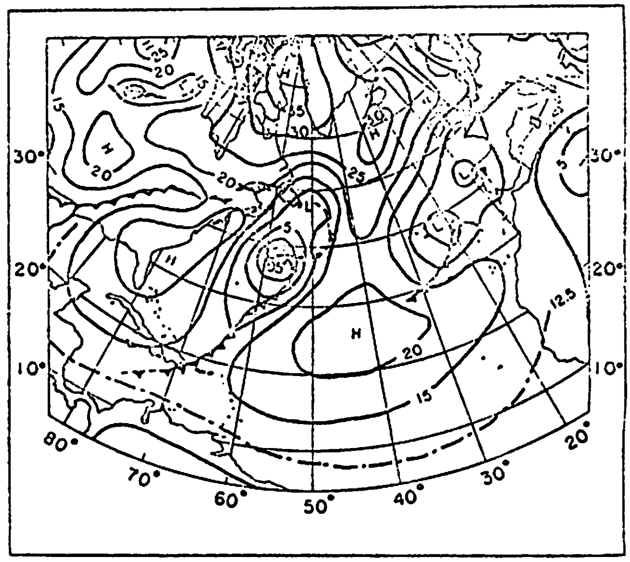


Fig. 49. A daily map showing the disturbance of the Bermuda–Azores high caused by a front pushing in from the northwest (Chase, 1951, MS).

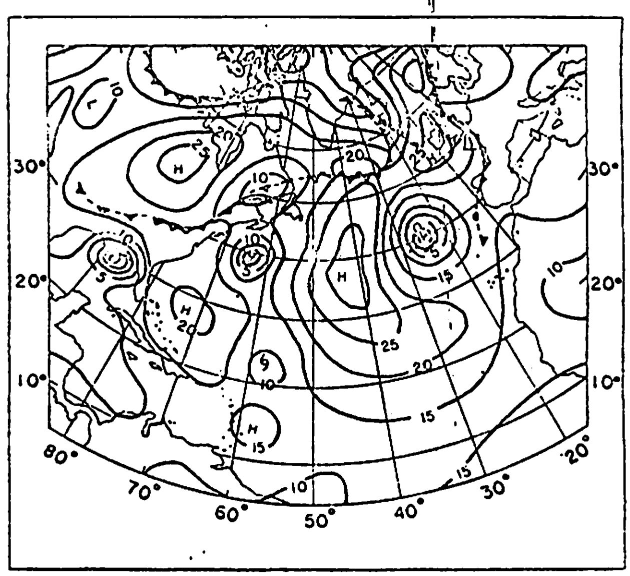


Fig. 50. A rare case of four simultaneous hurricanes over the North Atlantic (Chase, 1951, MS).

From Stommel (1960, p.79)

An approximation of the stress field caused by wind can be obtained from adding a wind term to the geostrophic equation

$$fV - \alpha \frac{\partial P}{\partial X} + \alpha \frac{\partial \tau x}{\partial z} = 0$$

Let x be in the direction of the surface wind and let Vg = the y component of the geostrophic wind

$$\frac{\partial \tau x}{\partial z} = - \rho f (Vg - V)$$

integrate from the surface z = 0 to a height z where $\tau x = 0$, so τo at the surface

$$\tau o = - f\rho \int_{o}^{z} (Vg-V)\, dz$$

Assuming the sea surface is isobaric. (Shepard and Omar, 1952)

This method apparently works well in the tropics (Shepard and Omar, 1952) but not so well in the unsteady westerlies (Shepard and others, 1952).

APPLICATION OF MODEL STUDIES OT THE ANTARCTIC CIRCUMPOLAR REGION

Returning to the fundamental problem of the correlation of models to actual circulation patterns, at first glance the Antarctic region is apparently an anomaly. The winds here seemingly swirl about Antarctica with unlimited fetch and as Munk and Palmen (1951) point out there are no obvious meridional barriers. Thus the circulation patterns should be strickly zonal as according to our models an ocean without meridional barriers can't have meridional transport caused by geostrophic components. This obviously is not the case as Antarctic waters in various water masses appear at the equator so there definitely is appreciable meridional transport occurring outside of the Ekman layers in the geostrophic portion of the Ocean.

One way to get around this difficulty is to employ some sort of lateral viscosity term. This solution suggested by the Japanese gives a lateral eddy viscosity of 10^{10} cm^2/sec just to limit the total circumpolar transport to 100 x $10^6 M^3$/sec. This is several orders of magnitude greater than predicted for the oceans. Addition of a vertical viscosity term is unrealistic because of the integrity of the Antarctic water masses such that the lateral term can not exceed 10^7 cm^2/sec or the vertical term 10 cm^2/sec.

Another reason for disregarding purely zonal flow is that observations of the flow of the circum Antarctic current show a southerly component which is incompatable with the northward deflection (to the left) of an easterly flowing current in the southern hemisphere.

Examination of cross-sections where surface flow is restricted or where there are narrow straits as between

(1) South America and the Palmer Penninsula

(2) Africa and Antarctica and

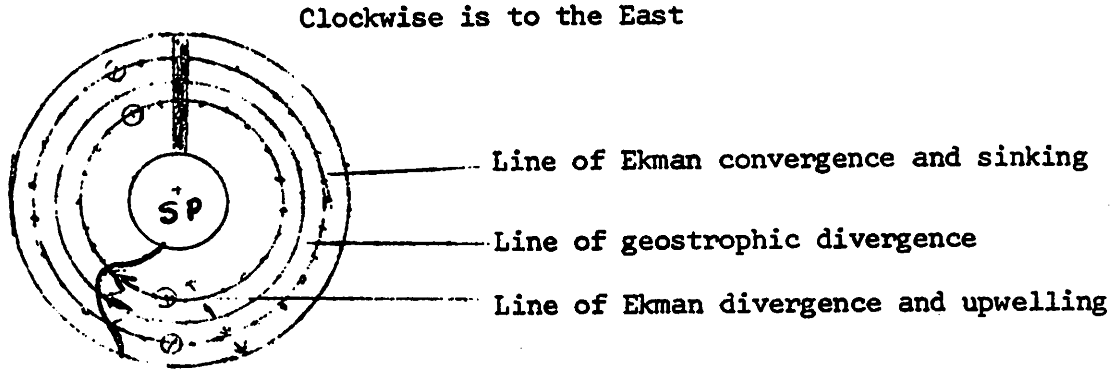
(3) Possibly between New Zealand and Antarctica

suggest the bottom topography may restrict the flow to the extent that these areas act as meridional barriers. This is particularly true between Cape Horn and the Palmer Peninsula where although the straits themselves are wide enough to permit adequate transport to the East, the Scotia Arch, a volcanic island chain, would be significant blocks to eastward transport driven by the prevailing westerly winds.

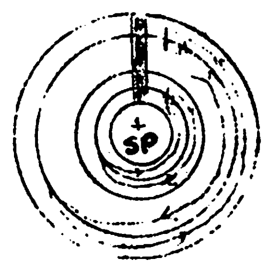
Barotropic
Baroclinic *

ZONAL WINDS - WITH ONE COMPLETE MERIDIONAL BARRIER

Clockwise is to the East

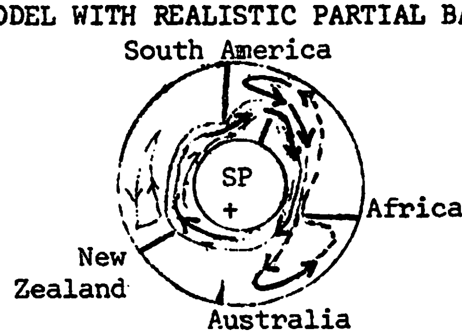


PRESSURE SYSTEMS AND TRANSPORT LINES



The southern most gyre around the low pressure system transports about $100 \times 10^6 M^3$/sec and the northern most somewhat less. Realistically the northern most gyre is interrupted by the protrusion of Africa and Asutralia-New Zealand Southward.

MODEL WITH REALISTIC PARTIAL BARRIERS



By breaking up the continuous barriek the transport line may become circum polar and the low pressure center shifts on to the Antarctic continent. Thus the apparent misfit is not so bad.

REFERENCES ON OCEAN MODELS

Bryan, K. and Cox, M. D., 1967, A Numerical Investigation of the Oceanic General Circulation: Tellus, V. 19, p. 54-80.

Niiler, P. P., 1966, On the Theory of Wind-driven Ocean Circulation: Deep-Sea Research, V. 13, p. 597-606.

Munk, W., 1966, Abyssal Recipes: Deep-Sea Research, V. 13, p. 707-730.

Stommel, H., 1957, A Survey of Ocean Current Theory: Deep-Sea Research, V. 4, p. 149-184.

________________, 1958, Abyssal Circulation: Deep-Sea Research, V. 5, p. 80-82.

________________, and Arons, A. B., 1960, On the Abyssal Circulation of the World Ocean I - Stationary Planetary Flow Patterns on a Sphere: Deep-Sea Research, V. 6, p. 140-154.

________________, 1960, On the Abyssal Circulation of the World Ocean II - An Idealized Model of Circulation Pattern and Amplitude in Oceanic Basins: Deep-Sea Research, V. 6, p. 217-233.

Arons, A. B., and Stommel, H., 1967, On the Abyssal Circulation of the World Ocean III - An Advection-Lateral Mixing Model of the Distribution of a Tracer Property in an Ocean Basin: Deep-Sea Research, V. 14, p. 441-457.

Veronis, G., 1966, Wind-Driven Circulation Part I, Linear Theory and Perturbation Analysis: Deep-Sea Research, V. 13, p. 17-29.

________________, 1966, Wind-Driven Circulation Part 2, Numerical Solutions of the Non-Linear Problem: Deep-Sea Research, V. 13, p. 31-55.

________________, 1966, Generation of Mean Ocean Circulation by Fluctuating Winds: Tellus, V. 18, p. 67-76.

Welander, P., 1966, A Two Layer Frictional Model of Wind-Driven Motion in a Rectangular Oceanic Basin: Tellus, V. 18, p. 54-62.

PEDLOSKY, J. AND GREENSPAN, H. P., 1967, A SIMPLE LABORATORY MODEL FOR THE OCEANIC CIRCULATION: JOUR. FLUID. MECH., V. 27 PART 2, p. 291-304.

KUO, H-H. AND VERONIS, G., 1970, DISTRIBUTION OF TRACERS IN THE DEEP OCEANS OF THE WORLD: DEEP-SEA. RES., V. 17, p. 29-46.

JOHNSON, J. A., 1970, OCEANIC BOUNDARY LAYERS: DEEP-SEA RES., V. 17, p. 455-465.